($\leftarrow$) It is obvious that any line segment between two points in C is included in C. One can construct a C-convex set S by $S = C$.

($\rightarrow$)

Notation: Define a curve similar to C and started from P to Q as C_{PQ}.

Let S the C-convex set, and let A, B two distinct points in S. For simplicity, assign a polar coordinate (r, θ) as $A = (0, 0)$, $B = (1, 0)$.
If C is not straight, there exist a point $X_1 = (p, \theta)(0 < p, 0 < \theta)$ which is in curve C_{AB} but not in segment $\overline{AB}$. By convexity, X_1 is in S.
(Note that p and θ are constants only dependent to shape of C.)

Define other point X_2 in curve C_{AX_1} s.t. ABX_1 is similar to AX_1X_2. X_2 has the coordinate $(p^2, 2\theta)$. Similarily, we define the consequence points $X_n = (p^n, n\theta)$ and they all are in S by convexity.

Choose n s.t. $\frac{4k+1}{2}\pi \leq n\theta \leq \frac{4k+3}{2}\pi$, then $|\overline{X_nB}| = 1 + p^{2n} - 2p^n \cos n\theta > |\overline{AB}|$.

Since p and θ are constants, for any two points (A, B) in S, we can find other two points (X_n, B) in S with constantly magnified length. Therefore, S cannot be bounded.

(Note: It is not sufficient to disprove the existence of "diameter"; open set does not have concrete diameter but may be bounded.)