POW2009-7

SUNGYOON KIM

Let α, β be $(x+\sqrt{x^2-1})^{1/n}$ and $(x-\sqrt{x^2-1})^{1/n}$, respectively. Assume that $q=\alpha+\beta$ is a rational number. Since $\alpha\beta=1$, α and β are two roots of $t^2-qt+1=0$. It is easy to see that a sequence $A_m=\alpha^m+\beta^m$ satisfies $A_{m+2}-qA_{m+1}+A_m=0$. Thus, we can write A_m as a linear combination of $A_0=2$ and $A_1=q$ with rational coefficients. This implies that A_m is rational. Therefore, $A_n=\alpha^n+\beta^n=2x$ is rational, or equivalently, x is rational, as desired.