First, note some basic properties of operator $*$:

$$a * b * \cdots * c + x = (a + x) * (b + x) * \cdots * (c + x)$$
$$x * x = x.$$

These can be easily proved.
For any rational number a and natural number n,

$$n(a * 0) = na * (n - 1)a * \cdots * a * 0,$$

which is proved using mathematical induction: suppose it is true for $n = k$, then

$$\begin{aligned}(k + 1)(a * 0) &= k(a * 0) + a * 0 \\ &= ka * (k - 1)a * \cdots * a * 0 + a * 0 \\ &= (ka + a * 0) * ((k - 1)a + a * 0) * \cdots * (a + a * 0) * (0 + a * 0) \\ &= ((k + 1)a * ka) * (ka * (k - 1)a) * \cdots * (2a * a) * (a * 0) \\ &= (k + 1)a * ka * (k - 1)a * \cdots * 2a * a * 0.\end{aligned}$$

Then,

$$\begin{aligned}n(a * 0) + na * 0 &= na * (n - 1)a * \cdots * a * 0 + na * 0 \\ &= 2na * (2n - 1)a * \cdots * a * 0 \\ &= 2n(a * 0)\end{aligned}$$

$\therefore na * 0 = n(a * 0)$. Substituting a by $\frac{1}{m}$, it is derived that $\frac{n}{m} * 0 = \frac{n}{m}(1 * 0)$, thus

$$q * 0 = q(1 * 0)$$

for any positive rational number $q = \frac{n}{m}$. Let $i = 1 * 0$.
Since $i = 1 * 0 = 1 * 1 * 0 = 1 * i$,

$$i = \begin{cases} 0 * (i - 1) + 1 = (i - 1)i + 1 & \text{if } i \geq 1 \\ (1 - i) * 0 + i = (i - 1)i + i & \text{if } i < 1 \end{cases}$$

$\therefore i = 1$ or 0. Thus, for any rational a and b where $a \geq b$:

$$a * b = (a - b) * 0 + b = \begin{cases} (a - b) + b = a = \max(a, b) & \text{if } i = 1 \\ b = \min(a, b) & \text{if } i = 0 \end{cases}$$