2017 International Workshop on Nonlinear PDE and Applications

March 30 - April 1, 2017

3221 Bldg 2E-1,
KAIST, Daejeon, Korea

Organizer : Soohyun Bae (Hanbat National University)
Jaeyoung Byeon (KAIST)
Yong-Jung Kim (KAIST)
Ki-Ahm Lee (SNU)
Yong-Hoon Lee (PNU)
Inbo Sim (University of Ulsan)

Program

March 30 Thursday

Session 1 Chairman : Soohyun Bae (Hanbat National University)

10:00-10:50 Yuki Naito, Ehime University
Separation structure of solutions for elliptic equations with exponential nonlinearity

10:50-11:10 Break

11:10-12:00 Yasuhito Miyamoto, The University of Tokyo
A limit equation and bifurcation diagrams of semilinear elliptic equations with general supercritical growth

12:00-14:00 Lunch

Session 2 Chairman : Yong-Hoon Lee (PNU)

14:00-14:50 Seick Kim, Yonsei Unviersity
On C^1, C^2, and weak type-$(1,1)$ estimates for linear elliptic operators

14:50-15:10 Break

15:10-16:00 Futoshi Takahashi, Osaka City University
Critical and subcritical fractional Trudinger-Moser type inequalities on $\mathbb{R}$

16:00-16:20 Break

16:20-17:10 Youngae Lee, NIMS
Degree counting for Toda system of rank two: one bubbling

18:00 Dinner

March 31 Friday

Session 1 Chairman : Ki-Ahm Lee (SNU)

09:30-10:20 Eiji Yanagida, Tokyo Institute of Technology
Extinction behavior of solutions of the logarithmic diffusion equation on $\mathbb{R}$

10:20-10:40 Break

10:40-11:30 Tongkeun Chang, Yonsei Unviersity
Nonhomogeneous initial-boundary value problems of the Stokes and the Navier-Stokes equations

11:30-11:50 Break

11:50-12:40 Toru Kan, Tokyo Institute of Technology
On the solution structure of a bistable reaction-diffusion equation on a thin dumbbell-shaped domain

12:40-14:30 Lunch

Session 2 Chairman : Jaeyoung Byeon (KAIST)

14:30-15:20 Sun-Sig Byun, SNU
Higher integrability results for a class of nonlinear elliptic and parabolic problems

15:20-15:40 Break

15:40-16:30 Rejeb Hadiji, University Paris-Est Créteil, UPEC
Minimization of Ginzburg-Landau energy with weight

16:30-16:50 Break

16:50-17:40 Jongmin Han, Kyung Hee University
String solutions for a graviataional Ginzburg-Landau model

18:30 Banquet

April 1 Saturday

Session 1 Chairman : Yong-Jung Kim (KAIST)

10:00-10:50 Soonsik Kwon, KAIST
Orbital stability of solitary waves for derivative nonlinear Schrödinger equations

10:50-11:10 Break

11:10-12:00 Woocheol Choi, KIAS
On the splitting method for the nonlinear Schrödinger equation with initial data in H^1

12:00-14:00 Lunch

Session 2 Chairman : Inbo Sim (University of Ulsan)

14:00-14:50 Ryuji Kajikiya, Saga University
Stability of stationary solutions for sublinear parabolic equations

14:50-15:10 Break

15:10-16:00 Namkwon Kim, Chosun University
Mixed type solutions in some Cherns-Simons gauge theory

16:00-16:20 Break

16:20-17:10 Satoshi Tanaka, Okayama University of Science
Symmetry-breaking bifurcation for the one-dimensional Hénon equation

18:00 Dinner

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KAIST BK21 Plus (Integration of Education and Research in Mathematical Science)

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Yuki Naito

Ehime University

Separation structure of solutions for elliptic equations with exponential nonlinearity

We consider radial solutions of the semilinear elliptic equation

$$\Delta u + K(|x|)e^u = 0 \quad \text{in } \mathbf{R}^N,$$

where $N \geq 3$, $K \in C(0,\infty)$, and $K(r) > 0$ for $r > 0$. We are interested in separation phenomena of radial solutions. In this talk, we first give a classification of the solution structures, and then we show some separation and intersection properties of solutions. In particular, we find that the equation changes its nature drastically according to the monotonicity of $K(r)$ when $N = 10$. This is a joint work with Professor Soohyun Bae (Hanbat National University).

Yasuhito Miyamoto

The University of Tokyo

A limit equation and bifurcation diagrams of semilinear elliptic equations with general supercritical growth

We study radial solutions of the semilinear elliptic equation $\Delta u + f(u) = 0$ under rather general growth conditions on f. We construct a radial singular solution and study the intersection number between the singular solution and a regular solution. Several applications of the intersection number are given: the Morse index of the singular solution, the bifurcation diagram of an elliptic Dirichlet problem in a ball, and the Type I blow-up solution of a parabolic problem. To this end, we derive a certain limit equation from the original equation at infinity, using a generalized similarity transformation.

Seick Kim

Yonsei Unviersity

On C^1, C^2, and weak type-$(1,1)$ estimates for linear elliptic operators

We show that any weak solution to elliptic equations in divergence form is continuously differentiable provided that the modulus of continuity of coefficients in the L^1-mean sense satisfies the Dini condition. This in particular answers a question recently raised by Yanyan Li and allows us to improve a result of Haim Brezis. We also prove a weak type-$(1,1)$ estimate under a stronger assumption on the modulus of continuity. The corresponding results for non-divergence form equations are also established.

Futoshi Takahashi

Osaka City University

Critical and subcritical fractional Trudinger-Moser type inequalities on $\mathbb{R}$

In this talk, we are concerned with the critical and subcritical Trudinger-Moser type inequalities for functions in a fractional Sobolev space on the whole real line. We prove the relation between two inequalities and discuss the attainability of the suprema.

Youngae Lee

NIMS

Degree counting for Toda system of rank two: one bubbling

In this talk, we study the degree counting formula of the rank two Toda system with simple singular sources. The key step is to derive the degree formula of the shadow system, which arises from the bubbling solutions as one of parameters crosses 4π. In order to compute the topological degree of the shadow system, we need to find some suitable deformation. During this deformation, we shall deal with new difficulty arising from the new phenomena: blow up does not necessarily imply concentration of mass. This phenomena occurs due to the collapsing of singularities. This talk is based on the joint works with Prof. Chang-Shou Lin, Prof. Juncheng Wei, Prof. Lei Zhang, and Dr. Wen Yang.

Eiji Yanagida

Tokyo Institute of Technology

Extinction behavior of solutions of the logarithmic diffusion equation on $\mathbb{R}$

We investigate the behavior of positive solutions to the Cauchy problem

$$\begin{cases} u_t = (\log u)_{xx}, & x \in \mathbb{R},\ t > 0, \\ \lim\limits_{x \to -\infty} (\log u)_x = \alpha, \quad \lim\limits_{x \to +\infty} (\log u)_x = -\beta, & t > 0, \\ u(x,0) = u_0(x), & x \in \mathbb{R}, \end{cases}$$

where α, β are given positive constants and $u_0(x)$ is a positive initial value. For this problem, due to the fast diffusion for small u, the extinction of solutions occurs in finite time, i.e., the solution vanishes at some $t = T < \infty$. The aim of this talk is to investigate precisely the behavior of solutions near the extinction time. By using an intersection number argument, it is shown that after rescaling of the time variable and the unknown variable, the solution approaches a certain profile which is given by a traveling pulse for the logarithmic diffusion equation with a linear source.

This is a joint work with Masahiko Shimojo (Okayama University of Science) and Peter Takáč (Universität Rostock).

Tongkeun Chang

Yonsei University

Nonhomogeneous initial-boundary value problems of the Stokes and the Navier-Stokes equations

In this talk, we introduce the solvability of the solution of the nonhomogeneous initial boundary value problem of Navier-Stokes equations;

$$\begin{cases} u_t - \Delta u + (u\cdot\nabla)u + \nabla p = 0 \quad \text{in} \quad \Omega\times(0,T), \\ div\, u = 0 \qquad \text{in} \quad \Omega\times(0,T), \\ u|_{\partial\Omega} = g, \quad u_{t=0} = u_0. \end{cases} \tag{0.1}$$

We will find the conditions of initial and boundary data for solvability of the solution. We want that the velocity u is contained in several kind of function spaces, for example $L^p(\Omega\times(0,T))$, $1 < p \le \infty$, weighted-$L^p(\Omega\times(0,T))$ or Holder space, etc. For this, we find the optimal conditions of initial and boundary data.

Toru Kan

Tokyo Institute of Technology

On the solution structure of a bistable reaction-diffusion equation on a thin dumbbell-shaped domain

On a thin dumbbell-shaped domain, we consider the Neumann problem of a bistable reaction-diffusion equation. As the thickness of the domain tends to zero, a limiting equation on a line segment appears. After introducing relationships between the original equation and the limiting equation, we discuss the solution structure of the limiting equation.

Sun-Sig Byun

SNU

Higher integrability results for a class of nonlinear elliptic and parabolic problems

We discuss some of recent improvements in the regularity theory of nonlinear elliptic and parabolic problems.

Rejeb Hadiji

University Paris-Est Créteil, UPEC

Minimization of Ginzburg-Landau energy with weight

Let Ω be a smooth bounded domain in $\mathbb{R}^2$. We consider the functional

$$E_\varepsilon(u) = \frac{1}{2}\int_\Omega f(x,u)\left|\nabla u\right|^2 + \frac{1}{4\varepsilon^2}\int_\Omega \left(1-|u|^2\right)^2$$

on the set $H^1_g(\Omega, \mathbf{C}) = \left\{u \in H^1(\Omega, \mathbf{C});\ u = g \text{ on } \partial\Omega\right\}$ where g is a smooth given boundary data with degree $d \geq 0$. In this talk, we consider the case where $f(x,u) = p_0 + |x|^k\,|u|^l$. We will study the behaviour of minimizers u_ε of E_ε and we will estimate the energy $E_\varepsilon(u_\varepsilon)$.

Jongmin Han

Kyung Hee University

String solutions for a graviataional Ginzburg-Landau model

In this talk, we introduce a Ginzburg-Landau model on a gravitational space-time. Under suitable hypothesis, we reduce the model on a two dimensional manifold and derive an elliptic equation on it for the energy minimizer. We exhibit recent results about radial solutions and multi-string solutions.

Soonsik Kwon

KAIST

Orbital stability of solitary waves for derivative nonlinear Schrödinger equations

We show the orbital stability of solitons arising in the cubic derivative nonlinear Schrodinger equations. We consider the endpoint case where the gauge transformed equation has zero mass. As opposed to other cases, this case enjoys L^2 scaling invariance. So we expect the orbital stability in the sense up to scaling symmetry, in addition to spatial and phase translations. For the proof, we are based on the variational argument and extend to a similar argument that was used for the proof of global existence for solutions with mass $< 4\pi$. Moreover, we also show a self- similar type blow up criteria of solutions with the critical mass 4π. This is a joint work with Yifei Wu.

Woocheol Choi

KIAS

On the splitting method for the nonlinear Schrödinger equation with initial data in H^1

In this work, we establish a convergence result for the operator splitting scheme Z_τ introduced by Ignat (10'), with initial data in H^1, for the nonlinear Schrödinger equation:

$$\partial_t u = i\Delta u + i\lambda |u|^p u, \qquad u(x,0) = \phi(x),$$

where $\lambda \in \{-1, 1\}$ and $(x,t) \in \mathbb{R}^d \times [0, \infty)$, with $0 < p < 4$ for $d = 3$ and $0 < p < \infty$ for $d = 1, 2$. This is based on the joint work with Youngwoo Koh.

Ryuji Kajikiya

Saga University

Stability of stationary solutions for sublinear parabolic equations

We study the stability of stationary solutions for a parabolic equation

$$\begin{aligned} u_t - \Delta u &= f(x,u) && \text{in } \Omega \times (0,\infty), \\ u &= 0 && \text{on } \partial\Omega \times (0,\infty), \\ u(x,0) &= u_0(x) && \text{in } \Omega, \end{aligned} \tag{0.2}$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$. We impose the next assumption.

Assumption 0.1. $f(x,u)$ is a continuous function on $\overline{\Omega} \times \mathbb{R}$ which is odd with respect to u, Hölder continuous with respect to u and satisfies

$$|f(x,u)| \le C(|u|^p + 1) \quad (u \in \mathbb{R},\ x \in \overline{\Omega}),$$

with some $C > 0$ and $p > 1$. Here we assume that $1 < p < \infty$ when $N = 1, 2$ and $1 < p < N/(N-2)$ when $N \ge 3$. For each $u \ne 0$, the second partial derivative $f_{uu}(x,u)$ exists and continuous on $\overline{\Omega} \times (\mathbb{R} \setminus \{0\})$. Let λ_1 be the first eigenvalue of the Laplacian. We assume that

$$f_u(x,u) < f(x,u)/u \quad (u > 0), \quad \limsup_{|u|\to\infty} (\max_{x\in\overline{\Omega}} f(x,u)/u) < \lambda_1,$$

$$\lim_{u\to 0} \left(\min_{x\in\overline{\Omega}} f_u(x,u) \right) = \infty.$$

Moreover there exists $L, u_0 > 0$, $\theta_0 \in (0,1)$ such that

$$|f_{uu}(x,v)| \le L|f_u(x,u)|/u + L/u, \quad (0 < u < u_0,\ v \in [(1-\theta_0)u, (1+\theta_0)u]).$$

Examples of $f(x,u)$ satisfying the assumption above are the following:

$$a(x)|u|^{p-1}u, \quad -a(x)u\log|u|, \quad a(x)|u|^{p-1}ue^{-|u|}, \quad a(x)\tanh(|u|^{p-1}u),$$

where $a(x) > 0$ is a continuous function and $0 < p < 1$. The next functional becomes a Lyapunov functional of (0.2):

$$E(u) := \int_\Omega \left(\frac{1}{2}|\nabla u|^2 - F(x,u) \right) dx, \quad F(x,u) := \int_0^u f(x,s)ds.$$

The stationary problem is written as

$$-\Delta v = f(x,v) \quad (x \in \Omega), \qquad v = 0 \quad (x \in \partial\Omega). \tag{0.3}$$

Proposition 0.2 (Known results). *(i) There exists a unique positive solution ϕ of* (0.3)*; moreover ϕ is a minimizer of E over $H_0^1(\Omega)$ and all minimizers of E consist only of $\pm\phi$.*

(ii) There exists a sequence v_n of non-trivial solutions for (0.3) *such that v_n converges to zero in $C^{1,\theta}(\overline{\Omega})$ as $n \to \infty$ for any $\theta \in (0,1)$.*

The stability is defined with respect to the $H_0^1(\Omega)$-norm. $\|u\|_p$ denotes the $L^p(\Omega)$ of u. We state the main results.

Theorem 0.3. *For any $u_0 \in H_0^1(\Omega)$,* (0.2) *has a bounded global solution in $H_0^1(\Omega)$. Its orbit is relatively compact. The ω limit set is a non-empty subset of the set of stationary solutions.*

Theorem 0.4. *There exists an $\varepsilon_0 > 0$ such that if v is a solution of* (0.3) *satisfying $\|v\|_\infty < \varepsilon_0$, then it is not asymptotically stable. Furthermore, if v is an isolated point of the set of stationary solutions, it is unstable. The zero solution is unstable.*

Theorem 0.5. *The positive stationary solution ϕ is exponentially stable. Moreover the exponent is the the first eigenvalue of the linearlized operator $-\Delta - f_u(x,\phi)$. Denote it by $\mu > 0$. For any $\varepsilon > 0$, there exists a $\delta > 0$ such that if $u(t)$ is a solution of* (0.2) *satisfying $\|u(0) - \phi\|_{H_0^1} < \delta$, then*

$$\|u(t) - \phi\|_{H_0^1} \leq \varepsilon e^{-\mu t} \quad (t \geq 0).$$

The exponent μ is optimal. Indeed, we have the theorem below.

Theorem 0.6. *Let $u_0 \in H_0^1(\Omega)$ satisfy either*

$$u_0(x) \geq (1+\delta_0)\phi(x) \quad or \quad 0 < u_0(x) \leq (1-\delta_0)\phi(x).$$

with some $\delta_0 \in (0,1)$. Then there exists a $c > 0$ such that a solution $u(t)$ with the initial data $u(0) = u_0$ satisfies

$$\|u(t) - \phi\|_{H_0^1} \geq \|u(t) - \phi\|_2 \geq ce^{-\mu t} \quad (t \geq 0).$$

Let $N = 1$, $\Omega = (0,1)$ and $f(x,u) \equiv f(u)$. For a non-negative integer k, we call a solution v of (0.3) a *k-nodal solution* if it has exactly k zeros in the interval $(0,1)$.

Theorem 0.7. *Let $N = 1$, $\Omega = (0, 1)$ and $f(x, u) \equiv f(u)$. Then for each $k \geq 1$, there exists a unique $(k-1)$-nodal solution v_k of (0.3) satisfying $v'(0) > 0$. The set of all solutions for (0.3) consists of $\pm v_k$ with $k \in \mathbb{N}$ and the zero solution. The positive stationary solution v_1 and the negative stationary solution $-v_1$ are exponentially stable with the exact exponent μ and other stationary solutions are unstable.*

Namkwon Kim

Chosun Unviersity

Mixed type solutions in some Cherns-Simons gauge theory

We show the existence of mixed type solutions in Chern-Simons theories of rank two over the entire space. Some estimates of mixed type solutions will be given first and the degree of the corresponding problem will be calculated. This is a joint work with K. Choe and C.S. Lin.

Satoshi Tanaka

Okayama University of Science

Symmetry-breaking bifurcation for the one-dimensional Hénon equation

This is a joint work with Professor Inbo Sim. In this talk, the problem for the one-dimensional Hénon equation

$$u'' + |x|^l u^p = 0, \quad u(-1) = u(1) = 0$$

is considered, where $l > 0$ and $p > 1$. The existence of a symmetry-breaking bifurcation point is proved. Moreover, the unbounded continuum, which emanates from the symmetry-breaking bifurcation point, is obtained. An example of a bounded branch connecting two symmetry-breaking bifurcation points is also given for the problem

$$u'' + |x|^{l(\lambda)} u^p = 0, \quad u(-1) = u(1) = 0,$$

where l is a specified continuous function.