

2nd International Workshop  
**Neumann-Poincaré Operator and Related Fields**

July 23-24, 2015  
KAIST, Daejeon, Korea

Organized by

Hyeonbae Kang (Inha University)  
Mikyong Lim (KAIST)  
Graeme Milton (University of Utah)

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BK21, Inha University, KMRS

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## Schedule

E2, Room 3221, KAIST

July 23

13:30-14:20, Habib Ammari (ENS, ETH)  
Mathematical modelling of plasmonic nanoparticles

14:30-15:00, Kazunori Ando (Inha Univ.)  
Plasmon resonance with finite frequencies

15:10-15:40, Sanghyeon Yu (KAIST)  
Spectral theory of the Neumann-Poincaré operator and cloaking by anomalous localized resonance for the elasto-static system

16:00-16:50, Mikyoung Lim (KAIST)  
Spectral resolution of the Neumann-Poincaré operator on intersecting disks and analysis of plasmon resonance

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July 24

10:00-11:00, Graeme W. Milton (Univ. of Utah)  
Bounds on the transient response of two-component composites (KMRS Intensive Lecture)

11:10-11:40, Owen Miller (MIT)  
Fundamental Limits to the Optical Response of Lossy Media

11:50-12:20, Aaron Welters (FIT)  
TBA

12:30-14:00, Lunch

14:00-14:50, Mihai Putinar (UC-Santa Barbara)  
Matrix positivity preservers in fixed dimension

## Abstracts

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Habib Ammari

Mathematical modelling of plasmonic nanoparticles

Localized surface plasmons are charge density oscillations confined to metallic nanoparticles. Excitation of localized surface plasmons by an electromagnetic field at an incident wavelength where resonance occurs results in a strong light scattering and an enhancement of the local electromagnetic fields. The aim of this talk is three-fold: (i) to analyze the shift and broadening of the plasmon resonance with changes in size and shape of the nanoparticles; (ii) to study the scattering and absorption enhancements by plasmon resonant nanoparticles; (iii) to show that one can achieve super-resolution and super-focusing using plasmonic nanoparticles. The presented results are from joint works with Pierre Millien, Matias Ruiz, and Hai Zhang.

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Kazunori Ando

Plasmon resonance with finite frequencies

We study resonance for the Helmholtz equation with a finite frequency in a plasmic material of negative dielectric constant in two and three dimensions. We show that the quasi-static approximation is valid for diametrically small inclusions. In fact, we quantitatively prove that if the diameter of an inclusion is small compared to the loss parameter, then resonance occurs exactly at eigenvalues of the Neumann-Poincaré operator associated with the inclusion; moreover, we show the blow-up rate of resonance. For the proof, we use the spectral theoretic approach.

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Mikyong Lim

Spectral resolution of the Neumann-Poincaré operator on intersecting disks and analysis of plasmon resonance

In this talk, we investigate the spectral nature of the Neumann-Poincaré operator on the intersecting disks, which is a domain with the Lipschitz boundary. The complete spectral resolution of the operator is derived, which shows in particular that it admits only the absolutely continuous spectrum, no singularly continuous spectrum and no pure point spectrum. We then quantitatively analyze using the spectral resolution the plasmon resonance at the absolutely continuous spectrum. This is a joint work with Hyeonbae Kang and Sanghyeon Yu.

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Owen Miller

Fundamental Limits to the Optical Response of Lossy Media

Metals interact strongly with electromagnetic radiation. They support subwavelength resonances that have generated interest in the field of "plasmonics" for more than a decade, but which are also damped by material loss. Here I will show that basic conservation-of-energy arguments lead to new, fundamental limits to the optical response of lossy media (including metals), independent of shape or periodicity. For quasistatic nanoparticles, we show that there are slightly smaller bounds for orientation or source averaging, either by sum rules derived via the Neumann-Poincare surface-integral operator or, equivalently, by a classical homogenization approach. The resulting optimal-extinction particles have been realized experimentally and improve on the state-of-the-art by an order of magnitude. For spontaneous emission rate-enhancement and near-field radiation, we show that the bounds are nearly tight in certain specialized cases, but that for many wavelengths the most common (unoptimized) structures are often orders of magnitude from the bounds, suggesting that dramatic future improvements may be possible with optimized geometries.

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Graeme W. Milton (with Ornella Mattei)

Bounds on the transient response of two-component composites

Bounds on the complex dielectric constant of a two-component material at fixed frequency were derived about 35 years ago independently by Milton and Bergman using the analytic representation formula for the effective dielectric constant as a function of the component dielectric constant. These bounds become tighter the more information is incorporated about the composite geometry, such as the volume fractions of the constituents and whether it is isotropic or not. These bounds were subsequently generalized to elasticity in works of Berryman, Gibiansky, Lakes and Milton, using the variational principles of Cherkaev and Gibiansky. All these bounds are applicable when the applied fields are time harmonic. But what happens when the applied fields are not time harmonic? One would like to bound for each moment in time, the transient response of the induced average displacement field given the applied time varying electric field. We obtain such bounds using the analytic method, and we find that they can be very tight, tighter the more information is known about the composite. The bounds are also applicable to the mathematically equivalent problem of antiplane elasticity, where one is interested in bounding the stress relaxation and creep of composites of two viscoelastic phases.

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Mihai Putinar

Matrix positivity preservers in fixed dimension

A celebrated 1942 result of Schoenberg characterizes all entry-wise functions which preserve positivity of matrices of any size. I will present a characterization of polynomials which preserve positivity when applied entry-wise on matrices of a fixed dimension. All put in historical context and motivated by recent demands of statistics of large data and optimization theory. A sketch of the proof will take a detour through the representation theory of the symmetric group.

Joint work with Alexander Belton, Dominique Guillot and Apoorva Khare.

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Sanghyeon Yu

Spectral theory of the Neumann-Poincaré operator and cloaking by anomalous localized resonance for the elasto-static system

We show that the general framework of the spectral theory of the Neumann-Poincaré (NP) operator for dielectric equations can be extended to that for the elasto-static system. However, there is a significant difference: the NP operator for the elasto-static system is not compact even if the domain on which the operator is defined is smooth. We show that the NP operators on disks, ellipses, and coated disks have point spectrums accumulating to some numbers determined by Lamé parameters. We then show on coated disks that cloaking by anomalous localized resonance takes place at accumulation points. This talk is based on joint papers with K. Ando, H. Kang and K. Kim.