

Research Highlights

Article

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Time-asymptotic stability of composite waves of viscous shock and rarefaction for barotropic Navier-Stokes equations, **Advances in Mathematics**, 2023 (419), 108963

Contents

We prove the time-asymptotic stability of composite waves composed of the superposition of a viscous shock and a rarefaction for the one-dimensional compressible barotropic Navier-Stokes equations. Our result solves a long-standing problem first mentioned in 1986 by Matsumura and Nishihara in [2]. The same authors introduced it officially as an open problem in 1992 in [3] and it was again described as very challenging open problem in 2018 in the survey paper [1]. The main difficulty is due to the incompatibility of the standard anti-derivative method, used to study the stability of viscous shocks, and the energy method used for the stability of rarefactions. Instead of the anti-derivative method, our proof uses the α -contraction with shifts theory recently developed by two of the authors. This method is energy based, and can seamlessly handle the superposition of waves of different kinds.

References

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