

Research Highlights

Article

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A Castelnuovo-Mumford regularity bound for threefolds with rational singularities,

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The famous *Eisenbud-Goto regularity conjecture* predicts that the Castelnuovo-Mumford regularity of an embedded projective complex variety X in P^r is bounded by $\deg(X) - \text{codim}(X) + 1$. Although McCullough-Peeva [MP] constructed counterexamples, it is widely believed that the regularity conjecture holds for mildly singular varieties. Following Lazarsfeld [L], to give a Castelnuovo-type bound on the regularity, one needs to consider the complexity of fibers of general projections to hypersurfaces. When X is smooth and has low dimension, the complexity of fibers of general projections is well understood. This gives a nice bound for the regularity of smooth surfaces [L] and smooth threefolds [K1, K2]. In this paper, we introduce new techniques using secant varieties and Loewy length to control the complexity of fibers of projections when X is singular. This leads us to give a nice bound for the regularity of normal threefolds with mild singularities (rational or Cohen-Macaulay Du Bois singularities).

References

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