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Vera Mikyoung Hur(UIUC)

Water waves: breaking, peaking and disintegration

Water waves describe the situation where water lies below a body of air and is acted upon by gravity. Describing what we may see or feel at the beach or in a boat, they are a prime example of applied mathematics. They encompass wide-ranging wave phenomena, from ripples driven by surface tension to tsunamis and to rogue waves. The interface between the water and the air is free and poses profound and subtle difficulties for rigorous analysis, numerical computation and modeling.

I will discuss some recent developments in the mathematical aspects of water wave phenomena. Particularly, (1) is the solution to the Cauchy problem regular, or do singularities form after some time? (2) are there solutions spatially periodic? (3) are they dynamically stable?

Juhi Jang(USC)

Global solutions to compressible Euler equations

I will discuss the global existence result for the three-dimensional compressible Euler equations illustrated by the stable expansion phenomenon. A virial identity argument indicates the growth of the fluid/gas support if it exists for all time. A finite family of explicit expanding solutions was recently constructed by Sideris through affine motions. I will review the virial identity argument and affine solutions. Then I will discuss how one can formulate the stability problem in the vacuum free boundary setting. The resulting PDEs can be viewed as a system of nonlinear wave equations in a bounded domain with degenerate coefficients. I will show that small perturbations lead to the global-in-time solutions by revealing the stabilizing effect of background expanding affine solutions. The lectures are based on joint works with Mahir Hadzic.

Kenji Nakanishi(RIMS)

Radial Strichartz, normal forms and global dynamics of nonlinear dispersive equations

Nonlinear dispersive equations are partial differential equations describing evolution of waves in various physical contexts, where the dynamics is governed by dispersion and nonlinear interactions of waves. Typical examples are the nonlinear Schrodinger equation and the KdV equation. Depending on the competition between the dispersion and the interactions, the solutions exhibit various types of behavior in space-time, such as scattering, solitons and blow-up. Recent developments in the mathematical analysis of those PDEs have enabled us to classify all possible behavior of solutions and to predict it from the initial data in some simple settings for model equations, where the global Strichartz estimates have played key roles, describing the dispersive nature of solutions by space-time integrability for linearized equations. In those model equations, however, nonlinear interactions tend to degenerate in high order for small amplitude of solutions, in order to avoid too strong competition with the dispersion. For more realistic equations with stronger interaction, the Strichartz estimate tends to be insufficient for us to control nonlinear solutions globally, and we often need much more elaborate multilinear estimates taking account of dispersion and interactions in detail. Nevertheless, in some cases, the Strichartz estimate turns out to be powerful for nonlinearity of lower order, if the solution is restricted by spherical symmetry or regularity, and if we combine it with certain quadratic transformations called normal forms. In this lecture, I will talk about the improved Strichartz estimates, the normal forms, and the global-in-time analysis of small and large solutions for the nonlinear dispersive equations, in particular the Zakharov system and the Gross-Pitaevskii equation.

Myeongju Chae(Hankyong National Univ)

On the global well-posedness of a parabolic-parabolic Keller-Segel equation

In this talk we consider the global well-posedness of the solutions to a parabolic-parabolic Keller-Segel equation in two dimensions. Mostly we work with a chemical consumption case. We also introduce a stability problem of the travelling wave solutions when a sensitivity function is logarithmic in the chemical concentration density.

Yonggeun Cho(Chonbuk National Univ)

Small data scattering for Boson star equation

In this talk we consider the small data scattering of boson star equations with Hartree type nonlinearity:

$$i\partial_t u + \Lambda u + (V * Q(u))u, u(0) = \varphi,$$

where $u : \mathbb{R}^{1+3} \to \mathbb{C}$ is the unknown wave function, $\Lambda = \sqrt{1-\Delta}$,

$$Q(u) = \lambda_0|u|^2 + \lambda_1 u^2 + \lambda_2 \overline{u}^2,$$

$V(x) = \frac{e^{-\mu_0|x|}}{|x|}(\mu_0 \geq 0)$ is the interaction potential. The equation has been studied to describe the dynamics and gravitational collapse of relativistic boson stars. This equation can also be referred to as a scalar version of 3D Dirac equation.

The scattering in $\mathcal{H}$ (a Hilbert space) means that any global solution u in time scatters to linear solutions $\varphi_\pm$ in the sense that

$$\|e^{-it\Lambda}u(t) - \varphi_\pm\|_{\mathcal{H}} \to 0 \text{ as } t \to \pm\infty.$$

When $\mu_0 = 0, \lambda_1 = \lambda_2 = 0$, it was shown by Cho and Ozawa (2006) that no scattering occurs in L^2. When $\mu_0 > 0, \lambda_1 = \lambda_2 = 0$, Herr and Tesfahun (2015) showed the small data scattering in H^s of radial solution for $s > 0$ and non-radial solution for $s > \frac{1}{2}$ by developing a hhl bilinear estimate. C. Yang considered the same problem and showed that the solution scatters in $H^{s,1}$ for $s > \frac{1}{4}$, where $H^{s,1}$ is a Sobolev type space taking in angular regularity with norm defined by $\|u\|_{H^{s,1}} = \|u\|_{H^s} + \|\mathbf{L}u\|_{H^s}$. He used new Strichartz estimates of Guo, Hani and Nakanishi (2018). Here $\mathbf{L}$ denotes the angular momentum operator and is defined by $x \times (-i\nabla)$. I believe that this result is almost sharp up to angular regularity. In this talk I am going to introduce the remaining case $\lambda_0 = 0$. We will see a scattering in H^s for any $s > 0$, which holds without resort to angular regularity. The methods are non-resonance interaction between two waves and $U^p - V^p$ space argument.

Tatsuo Iguchi(Keio)

Isobe-Kakinuma model for water waves as a higher order shallow water approximation

We consider the initial value problem to the Isobe-Kakinuma model for water waves. As was shown by J. C. Luke, the water wave problem has a variational structure. By approximating the velocity potential in Luke's Lagrangian, we obtain an approximate Lagrangian for water waves. The Isobe-Kakinuma model is a corresponding Euler-Lagrange equation for the approximate Lagrangian. In this talk, we first explain a structure of the Isobe-Kakinuma model and then justify the model rigorously as a higher order shallow water approximation by giving an error estimate between the solutions of the model and of the full water wave problem. It is revealed that the Isobe-Kakinuma model is a much more precise model than the well-known Green-Naghdi equations.

Donghyun Lee(POSTECH)

The Boltzmann equation in bounded domains

The Boltzmann equation is a mathematical model for rarefied gas. We study basic properties of the equation and also impose several boundary conditions. We also study recent results and main ideas for global existence and asymptotic behaviors (Maxwellian, equilibrium state).