“Nonlinear Modulational Stability of Periodic Traveling-Wave Solutions of the Generalized Kuramoto-Sivashinsky Equation” (Barker, Johnson, Noble, Rodrigues, Zumbrun — 2013)

The goal of covering this section is to introduce participants to the stability of periodic patterns with a concrete (although non-trivial) example. Sections to cover are…

Appendix A.1, getting to Proposition A.2 — Deals with existence of periodic traveling waves in the KdV/KS equation.

Section 1 — General Intro, main result, presents basics of Bloch theory / periodic spectral theory.

Section 3 — Proof of main result Theorem 1.1.

Appendix B — This deals with the analogous stability result for the Swift-Hohenberg equation, which is a non-conservative counterpart to the KdV/KS equation. The stability analysis is easier here since the associated linearized operator (at zero Bloch-frequency) has a simple kernel, where as for the KdV/KS equation it has a one-dimensional kernel with a one-dimensional Jordan block.

"Non-Localized Modulation of Periodic Reaction Diffusion Waves: Nonlinear Stability” (Johnson, Noble, Rodrigues, Zumbrun — 2013)

The goal of covering this section is to show how the theory from the first paper may be extended to consider perturbations that are NOT localized, consisting rather of a localized piece plus an asymptotic phase shift at spatial \pm\infty. That is, what if we take a periodic wave and shift it slightly out of phase at \pm\infty? Will it relax back to its original form? If so, how will this happen?

I think it would be good to carry out this theory for the Swift-Hohenberg (SH) equation. While this is not specifically addressed in this paper, the techniques can be adapted and applied to SH. I am happy to write up all the details and work with participants on the this. The relevant details in this paper would be Section 4 (how to deal with “modulational initial data”) and Section 8 (details of how to refine the stability analysis discussed above to accommodate for this more general setting).

Modulational Instability in Equations of KdV Type” (Bronski, Hur, Johnson — 2016)

In here, there is a general theory for analyzing such problems via spectral perturbation theory contained in Sections 4.1-4.3. Also, the theory is “easier” in for small amplitude waves where one can use asymptotic expansions of solutions in addition to spectral perturbation theory