

# Contents

<i>Preface</i>	v
1. Introduction	1
2. Chapter 2. Manifolds and $\mathcal{G}$ -structures	5
2.1 The review of topology . . . . .	5
2.1.1 Manifolds . . . . .	5
2.1.2 Some homotopy theory . . . . .	7
2.1.3 Covering spaces and discrete group actions . . . . .	8
2.1.4 Simplicial manifolds . . . . .	10
2.2 Lie groups . . . . .	14
2.2.1 Manifolds and tangent spaces . . . . .	14
2.2.2 Lie groups . . . . .	14
2.2.3 Lie algebras . . . . .	15
2.2.4 Lie groups and Lie algebras . . . . .	16
2.2.5 Lie group actions . . . . .	17
2.3 Pseudo-groups and $\mathcal{G}$ -structures . . . . .	18
2.3.1 $\mathcal{G}$ -manifolds . . . . .	19
2.4 Differential geometry . . . . .	20
2.4.1 Riemannian manifolds . . . . .	20
2.4.2 Principal bundles and connections: flat connections . . . . .	21
2.5 Notes . . . . .	24
3. Chapter 3. Geometry and discrete groups	25
3.1 Geometries . . . . .	25
3.1.1 Euclidean geometry . . . . .	25
3.1.2 Spherical geometry . . . . .	26
3.1.3 Affine geometry . . . . .	27
3.1.4 Projective geometry . . . . .	27
3.1.5 Conformal geometry . . . . .	32

3.1.6	Hyperbolic geometry . . . . .	34
3.1.7	Models of hyperbolic geometry . . . . .	36
3.2	Discrete groups and discrete group actions . . . . .	40
3.2.1	Convex polyhedrons . . . . .	41
3.2.2	Convex polytopes . . . . .	42
3.2.3	The fundamental domains of discrete group actions . . . . .	43
3.2.4	Side pairings and the Poincaré fundamental polyhedron theorem . . . . .	44
3.2.5	Crystallographic groups . . . . .	50
3.3	Notes . . . . .	50
4.	Chapter 4. Topology of orbifolds	51
4.1	Compact group actions . . . . .	51
4.1.1	Tubes and slices . . . . .	53
4.1.2	Locally smooth actions . . . . .	55
4.1.3	Manifolds as quotient spaces. . . . .	55
4.1.4	Smooth actions are locally smooth . . . . .	56
4.1.5	Equivariant triangulation . . . . .	57
4.2	The definition of orbifolds . . . . .	57
4.2.1	Local groups and the singular set . . . . .	59
4.2.2	Examples: good orbifolds . . . . .	62
4.2.3	Examples: silvering . . . . .	64
4.3	The definition as a groupoid . . . . .	66
4.3.1	Groupoids . . . . .	67
4.3.2	An abstract definition . . . . .	68
4.4	Differentiable structures on orbifolds . . . . .	70
4.4.1	Bundles over orbifolds . . . . .	71
4.4.2	Tangent bundles and tensor bundles . . . . .	72
4.4.3	The existence of a locally finite good covering . . . . .	75
4.4.4	Silvering the boundary components . . . . .	75
4.4.5	The Gauss-Bonnet theorem . . . . .	75
4.5	Triangulation of smooth orbifolds . . . . .	76
4.5.1	Triangulation of the stratified spaces . . . . .	76
4.5.2	Orbifolds as stratified spaces . . . . .	79
4.6	Covering spaces of orbifolds . . . . .	82
4.6.1	The fiber product construction by Thurston . . . . .	83
4.6.2	Universal covering orbifolds by fiber-products . . . . .	85
4.7	The path-approach to the universal covering spaces following Haefliger . . . . .	91
4.7.1	$\mathcal{G}$ -paths . . . . .	91
4.7.2	Covering spaces and the fundamental group . . . . .	95
4.8	Notes . . . . .	96

5.	Chapter 5. Topology of 2-orbifolds	97
5.1	The properties of 2-orbifolds . . . . .	97
5.1.1	The triangulation of 2-orbifolds . . . . .	99
5.1.2	The classification of 1-dimensional suborbifolds of 2-orbifolds	99
5.1.3	The orbifold Euler-characteristic for orbifolds due to Satake	100
5.1.4	The generalized Riemann-Hurwitz formula . . . . .	101
5.1.5	A geometrization of 2-orbifolds : a partial result . . . . .	102
5.1.6	Good, very good, and bad 2-orbifolds . . . . .	103
5.2	Topological operations on 2-orbifolds: constructions and decompositions . . . . .	105
5.2.1	The definition of splitting and sewing 2-orbifolds . . . . .	106
5.2.2	Regular neighborhoods of 1-orbifolds . . . . .	107
5.2.3	Splitting and sewing on 2-orbifolds reinterpreted . . . . .	109
5.2.4	Identification interpretations of splitting and sewing . . . . .	110
5.3	Notes . . . . .	111
6.	Chapter 6. Geometry of orbifolds: geometric structures on orbifolds	113
6.1	The definition of geometric structures on orbifolds . . . . .	113
6.1.1	An atlas of charts approach . . . . .	113
6.1.2	The developing maps and holonomy homomorphisms . . . . .	115
6.1.3	The definition as flat bundles with transversal sections . . . . .	115
6.1.4	The equivalence of three notions. . . . .	117
6.2	The definition of the deformation spaces of $(G, X)$ -structures on orbifolds . . . . .	117
6.2.1	The isotopy-equivalence space. . . . .	117
6.2.2	The topology of the deformation space . . . . .	118
6.2.3	The local homeomorphism theorem . . . . .	118
6.3	Notes . . . . .	120
7.	Chapter 7. Deformation spaces of hyperbolic structures on 2-orbifolds	121
7.1	The definition of the Teichmüller space of 2-orbifolds . . . . .	121
7.2	The geometric cutting and pasting and the deformation spaces . . . . .	122
7.2.1	Geometric constructions. . . . .	122
7.3	The decomposition of 2-orbifolds into elementary 2-orbifolds. . . . .	124
7.3.1	Elementary 2-orbifolds. . . . .	126
7.4	The Teichmüller spaces for 2-orbifolds . . . . .	128
7.4.1	The strategy of the proof . . . . .	128
7.4.2	The generalized hyperbolic triangle theorem . . . . .	128
7.4.3	The proof of Proposition 7.4.1. . . . .	129
7.4.4	The steps to prove Theorem 7.0.1. . . . .	130
7.5	Notes . . . . .	132

8.	Chapter 8. Deformation spaces of real projective structures on 2-orbifolds	133
8.1	Introduction to real projective orbifolds . . . . .	133
8.1.1	Examples of real projective 2-orbifolds. . . . .	136
8.2	A survey of real projective structures on surfaces of negative Euler characteristic. . . . .	143
8.2.1	Topological work . . . . .	145
8.2.2	The gauge theory and projective structures. . . . .	145
8.2.3	Hitchin's conjecture and the generalizations. . . . .	145
8.2.4	Group theory and representations . . . . .	147
8.3	Real projective structures on 2-orbifolds of negative Euler characteristic. . . . .	148
8.3.1	Real projective 2-orbifolds and the Hitchin-Teichmüller components . . . . .	150
8.3.2	Understanding the deformation space of real projective structures . . . . .	152
8.4	Notes . . . . .	159
	<i>Bibliography</i>	163
	<i>Index</i>	169