Logic and the set theory

Lecture 5: Propositional Calculus: part 1

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About this lecture

- Notions of Inference

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- Inference Rules

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- Hypothetical Rules

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- Grading and so on in the moodle. Ask questions in moodle.

Some helpful references

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- `http://plato.stanford.edu/contents.html` has much resource. See "Realism, Informal logic 2. Deductivism and beyond," and "Nondeductive methods in mathematics."

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- `http://ocw.mit.edu/OcwWeb/Linguistics-and-Philosophy/24-241Fall-2005/CourseHome/` See "Derivations in Sentential Calculus". (or SC Derivations.)

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- `http://jvrosset.free.fr/Goedel-Proof-Truth.pdf` “Does Godels incompleteness prove that truth transcends proof?”

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- http://svn.oriontransfer.org/TruthTable/index.rhtml, has xor, complete.

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- Realism believes in the existence and the independence of certain objects and so on. This is very close to the logical atomism.
- Antirealism: One has to test to find out before it can considered to exists and so on.
- Since we do not know everything, which should we take as our position?

The notion of inference

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- This is actually weaker than TF table or truth tree method. (in higher-order logic)
- If you take the antirealist's position, the deductions are only valid method. But we could also take the realist's position.
- The reason for doing it is that for Predicate calculus, TF methods cannot work since we have to check infinitely many cases. (incompleteness)

Nonhypothetical Inference Rules

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- $P, P \rightarrow Q, \vdash Q$.
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- Example:
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- We need two ($\rightarrow E$).

More rules

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- Biconditional introduction.($\leftrightarrow I$): $\phi \rightarrow \psi$, $\psi \rightarrow \phi$. Then $\phi \leftrightarrow \psi$.

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- Biconditional elimination. ($\leftrightarrow E$): $\phi \leftrightarrow \psi$. Then $\phi \to \psi$, $\psi \to \phi$.

Example 1

- $P \vdash (P \vee Q) \wedge (P \vee R)$.

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- 1. P. Assumption
- 2. $P \vee Q$. 1. $\vee I$.
- 3. $P \vee R$. 1. $\vee I$.
- 4. $(P \vee Q) \wedge (P \vee R)$. 2.3. $\wedge I$.

Example 2

- $P, \neg\neg(P \to Q) \vdash (R \wedge S) \vee Q$.

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- $P, \neg\neg(P \rightarrow Q) \vdash (R \wedge S) \vee Q$.
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- $P, \neg\neg(P \to Q) \vdash (R \land S) \lor Q$.
- 1. P. Assumption
- 2. $\neg\neg(P \to Q)$. A.
- 3. $P \to Q$. 2. $\neg E$.

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- 2. $\neg\neg(P \to Q)$. A.
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- 5. $(R \wedge S) \vee Q$ 4. $\vee I$.

Example 3

- $P \vee P, P \rightarrow (Q \wedge R) \vdash R$.

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- $P \vee P, P \to (Q \wedge R) \vdash R$.
- 1. $P \vee P$. A.

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- $P \vee P, P \rightarrow (Q \wedge R) \vdash R$.
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- 2. $P \rightarrow (Q \wedge R)$. A.
- 3. $Q \wedge R$. 1, 2. $\vee E$.

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- $P \vee P, P \rightarrow (Q \wedge R) \vdash R$.
- 1. $P \vee P$. A.
- 2. $P \rightarrow (Q \wedge R)$. A.
- 3. $Q \wedge R$. 1, 2. $\vee E$.
- 4. R. 3. $\wedge E$.

Hypothetical Rules

- Conditional introduction ($\to I$): Given a derivation of ϕ with help of ψ, we infer $\psi \to \phi$.

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- : Q. 1,3, $\rightarrow E$.

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- $Q \rightarrow R$. A.
- : P. H.
- : Q. 1,3, $\rightarrow E$.
- : R. 2.4, $\rightarrow E$.

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- Example:
- $P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$. (Socrates is human, Humans are mortal, Thus, Socrates is mortal.)
- $P \rightarrow Q$. A.
- $Q \rightarrow R$. A.
- : P. H.
- : Q. 1,3, $\rightarrow E$.
- : R. 2.4, $\rightarrow E$.
- $P \rightarrow R$. 3-5. $\rightarrow I$.

Example

- $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$.

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- 1. $(P \wedge Q) \vee (P \wedge R)$. A

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- 1. $(P \wedge Q) \vee (P \wedge R)$. A
- 2. : $P \wedge Q$. H

Example

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- 1. $(P \wedge Q) \vee (P \wedge R)$. A
- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.

Example

- $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$.
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- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.

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- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
- 5. : $Q \vee R$. 4. $\vee I$.

Example

- $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$.
- 1. $(P \wedge Q) \vee (P \wedge R)$. A
- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
- 5. : $Q \vee R$. 4. $\vee I$.
- 6 : $P \wedge (Q \vee R)$. 3.5. $\wedge I$.

Example

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- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
- 5. : $Q \vee R$. 4. $\vee I$.
- 6 : $P \wedge (Q \vee R)$. 3.5. $\wedge I$.
- 7. $(P \wedge Q) \rightarrow (P \wedge (Q \vee R))$. 2-6 $\rightarrow I$.

Example

- $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$.
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- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
- 5. : $Q \vee R$. 4. $\vee I$.
- 6 : $P \wedge (Q \vee R)$. 3.5. $\wedge I$.
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- 8. : $P \wedge R$. H

Example

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- 1. $(P \wedge Q) \vee (P \wedge R)$. A
- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
- 5. : $Q \vee R$. 4. $\vee I$.
- 6 : $P \wedge (Q \vee R)$. 3.5. $\wedge I$.
- 7. $(P \wedge Q) \rightarrow (P \wedge (Q \vee R))$. 2-6 $\rightarrow I$.
- 8. : $P \wedge R$. H
- 9. : P 8. $\wedge E$.

Example

- $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$.
- 1. $(P \wedge Q) \vee (P \wedge R)$. A
- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
- 5. : $Q \vee R$. 4. $\vee I$.
- 6 : $P \wedge (Q \vee R)$. 3.5. $\wedge I$.
- 7. $(P \wedge Q) \rightarrow (P \wedge (Q \vee R))$. 2-6 $\rightarrow I$.
- 8. : $P \wedge R$. H
- 9. : P 8. $\wedge E$.
- 10. : R 8. $\wedge E$.

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- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
- 5. : $Q \vee R$. 4. $\vee I$.
- 6 : $P \wedge (Q \vee R)$. 3.5. $\wedge I$.
- 7. $(P \wedge Q) \rightarrow (P \wedge (Q \vee R))$. 2-6 $\rightarrow I$.
- 8. : $P \wedge R$. H
- 9. : P 8. $\wedge E$.
- 10. : R 8. $\wedge E$.
- 11. : $Q \vee R$. 10. $\vee I$.

Example

- $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$.
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- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
- 5. : $Q \vee R$. 4. $\vee I$.
- 6 : $P \wedge (Q \vee R)$. 3.5. $\wedge I$.
- 7. $(P \wedge Q) \to (P \wedge (Q \vee R))$. 2-6 $\to I$.
- 8. : $P \wedge R$. H
- 9. : P 8. $\wedge E$.
- 10. : R 8. $\wedge E$.
- 11. : $Q \vee R$. 10. $\vee I$.
- 12 : $P \wedge (Q \vee R)$. 9.11. $\wedge I$.

Example

- $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$.
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- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
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- 7. $(P \wedge Q) \rightarrow (P \wedge (Q \vee R))$. 2-6 $\rightarrow I$.
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- 9. : P 8. $\wedge E$.
- 10. : R 8. $\wedge E$.
- 11. : $Q \vee R$. 10. $\vee I$.
- 12 : $P \wedge (Q \vee R)$. 9.11. $\wedge I$.
- 13. $(P \wedge R) \rightarrow (P \wedge (Q \vee R))$. 2-6 $\rightarrow I$.

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- 12 : $P \wedge (Q \vee R)$. 9.11. $\wedge I$.
- 13. $(P \wedge R) \rightarrow (P \wedge (Q \vee R))$. 2-6 $\rightarrow I$.
- 14. $P \wedge (Q \vee R)$. 1.7.13 $\vee E$.

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- If two or more hypotheses are ineffect, then the order that they are discharged is reversed.
- A proof is not valid until all the hypotheses are discharged.

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  - 2. : $\neg P$. H (for $\neg I$)

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Example

- $P \to Q \vdash \neg P \vee Q$.

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- $P \to Q \vdash \neg P \vee Q$.
- 1. $P \to Q$.
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- $P \rightarrow Q \vdash \neg P \vee Q$.
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- 2. : $\neg(\neg P \vee Q)$. H (for $\neg I$.)
- 3. ::P. H.(for $\neg I$).
- 4. :: Q. 1.3. ($\rightarrow E$)
- 5. :: $\neg P \vee Q$. 4 $\vee I$.

Example

- $P \to Q \vdash \neg P \vee Q$.
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- 2. : $\neg(\neg P \vee Q)$. H (for $\neg I$.)
- 3. ::P. H.(for $\neg I$).
- 4. :: Q. 1.3. $(\to E)$
- 5. :: $\neg P \vee Q$. 4 $\vee I$.
- 6. :: $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2.5. $\wedge I$.

Example

- $P \to Q \vdash \neg P \vee Q$.
- 1. $P \to Q$.
- 2. : $\neg(\neg P \vee Q)$. H (for $\neg I$.)
- 3. ::P. H.(for $\neg I$).
- 4. :: Q. 1.3. $(\to E)$
- 5. :: $\neg P \vee Q$. 4 $\vee I$.
- 6. :: $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2.5. $\wedge I$.
- 7. : $\neg P$. 3-6 $\neg I$.

Example

- $P \to Q \vdash \neg P \lor Q$.
- 1. $P \to Q$.
- 2. : $\neg(\neg P \lor Q)$. H (for $\neg I$.)
- 3. ::P. H.(for $\neg I$).
- 4. :: Q. 1.3. ($\to E$)
- 5. :: $\neg P \lor Q$. 4 $\lor I$.
- 6. :: $(\neg P \lor Q) \land \neg(\neg P \lor Q)$. 2.5. $\land I$.
- 7. : $\neg P$. 3-6 $\neg I$.
- 8. : $\neg P \lor Q$. 7. $\lor I$.

Example

- $P \to Q \vdash \neg P \vee Q$.
- 1. $P \to Q$.
- 2. : $\neg(\neg P \vee Q)$. H (for $\neg I$.)
- 3. ::P. H.(for $\neg I$).
- 4. :: Q. 1.3. ($\to E$)
- 5. :: $\neg P \vee Q$. 4 $\vee I$.
- 6. :: $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2.5. $\wedge I$.
- 7. : $\neg P$. 3-6 $\neg I$.
- 8. : $\neg P \vee Q$. 7. $\vee I$.
- 9. : $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2. 8 $\wedge I$.

Example

- $P \to Q \vdash \neg P \vee Q$.
- 1. $P \to Q$.
- 2. : $\neg(\neg P \vee Q)$. H (for $\neg I$.)
- 3. ::P. H.(for $\neg I$).
- 4. :: Q. 1.3. ($\to E$)
- 5. :: $\neg P \vee Q$. 4 $\vee I$.
- 6. :: $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2.5. $\wedge I$.
- 7. : $\neg P$. 3-6 $\neg I$.
- 8. : $\neg P \vee Q$. 7. $\vee I$.
- 9. : $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2. 8 $\wedge I$.
- 10. $\neg\neg(\neg P \vee Q)$. 2-9 $\neg I$.

Example

- $P \to Q \vdash \neg P \vee Q$.
- 1. $P \to Q$.
- 2. : $\neg(\neg P \vee Q)$. H (for $\neg I$.)
- 3. ::P. H.(for $\neg I$).
- 4. :: Q. 1.3. ($\to E$)
- 5. :: $\neg P \vee Q$. 4 $\vee I$.
- 6. :: $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2.5. $\wedge I$.
- 7. : $\neg P$. 3-6 $\neg I$.
- 8. : $\neg P \vee Q$. 7. $\vee I$.
- 9. : $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2. 8 $\wedge I$.
- 10. $\neg\neg(\neg P \vee Q)$. 2-9 $\neg I$.
- 11. $\neg P \vee Q$. 10. $\neg E$.