Logic and the set theory Lecture 17: Functions in How to Prove It.

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Functions

- Functions
- One to one and onto

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- Inverse of functions

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- Functions
- One to one and onto
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- Images and inverse images

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- Course homepages: http://mathsci.kaist.ac.kr/~schoi/logic.html and the moodle page http://KLMS.kaist.ac.kr

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- Course homepages: http://mathsci.kaist.ac.kr/~schoi/logic.html and the moodle page http://KLMS.kaist.ac.kr
- Grading and so on in the moodle. Ask questions in moodle.

Image: Image:

Some helpful references

• Sets, Logic and Categories, Peter J. Cameron, Springer. Read Chapters 3,4,5.

Image: A matrix

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- http://plato.stanford.edu/contents.html has much resource.
- Introduction to set theory, Hrbacek and Jech, CRC Press. (Chapter 2)

• Let *F* be a relation from *A* to *B*. *F* is said to be a *function* from *A* to *B* if $\forall a \in A \exists ! b \in B((a, b) \in F).$

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- Examples:

 $f = \{(s, p) | \text{ Prof p is the advisor of the student s } \}.$

Theorem

 $f, g : A \rightarrow B$. If $\forall a \in A(f(a) = g(a))$, then f = g.

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Proof.

Given f(a) = g(a) for all $a \in A$, we show $f \subset g$ and $g \subset f$. Given $\forall a \in A$, f(a) = g(a), we show $(a, b) \in f \to (a, b) \in g$ first. But (a, b = f(a)) = (a, b = g(a)). Thus clear. We show $(a, b) \in g \to (a, b) \in f$ similarly.

Composition

• $Ran(f) = \{f(a) | a \in A\}$. Dom(f) = A.

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- Given $f : A \rightarrow B$ and $g : B \rightarrow C$, we define $g \circ f$ as relation.

Composition

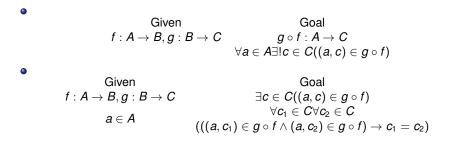
- $Ran(f) = \{f(a) | a \in A\}$. Dom(f) = A.
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Theorem

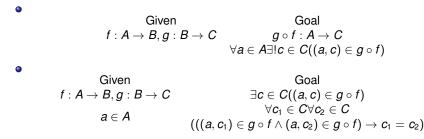
Let $f : A \rightarrow B, g : B \rightarrow C$. Then $g \circ f : A \rightarrow C$ and for all $a \in A, g \circ f(a) = g(f(a))$.

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• Existence part. clear.

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• Uniqueness part:

• Uniqueness part:

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$$\begin{array}{ccc} \text{Given} & \text{Goal} \\ f: A \rightarrow B, g: B \rightarrow C & c_1 = c_2 \\ a \in A & \\ c_1, c_2 \in C \\ (a, c_1) \in g \circ f, (a, c_2) \in g \circ f \end{array}$$

• Uniqueness part:

Given Goal $f: A \rightarrow B, g: B \rightarrow C$ $c_1 = c_2$ $a \in A$ $c_1, c_2 \in C$ $(a, c_1) \in g \circ f, (a, c_2) \in g \circ f$

• $(a, b_1) \in f, (b_1, c_1) \in g, (a, b_2) \in f, (b_2, c_2) \in g$. Here $b_1 = b_2$ and hence $c_1 = c_2$.

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One to one and onto

Definition

• $f: A \rightarrow B$ is said to be *one-to-one* if

$$\neg \exists a_1 \in A \exists a_2 \in A(f(a_1) = f(a_2) \land a_1 \neq a_2).$$

• *f* is onto if $\forall b \in B \exists a \in A(f(a) = b)$.

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Theorem

- *f* is one-to-one iff $\forall a_1 \in A \forall a_2 \in A(f(a_1) = f(a_2) \rightarrow a_1 = a_2)$.
- f is onto iff Ran(f) = B.

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Theorem

- *f* is one-to-one iff $\forall a_1 \in A \forall a_2 \in A(f(a_1) = f(a_2) \rightarrow a_1 = a_2)$.
- f is onto iff Ran(f) = B.

Theorem

- Let $f : A \rightarrow B, g : B \rightarrow C, g \circ f : A \rightarrow C$.
 - If f and g are both one-to-one, then so is $g \circ f$.
 - If f and g are both onto, then so is $g \circ f$.

• Injectivity only:

 $\begin{array}{c} \text{Given} \\ \forall a_1 \in B \forall a_2 \in B \\ ((f(a_1) = f(a_2) \rightarrow a_1 = a_2) \\ \forall b_1 \in B \forall b_2 \in B \\ ((g(b_1) = g(b_2) \rightarrow b_1 = b_2) \end{array}$

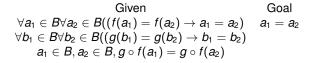
$$\begin{array}{c} \text{Goal} \\ \forall a_1 \in B \forall a_2 \in B \\ ((g \circ f(a_1) = g \circ f(a_2) \rightarrow a_1 = a_2) \end{array}$$

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• Injectivity only:

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• Injectivity only:

 $\begin{array}{ll} \text{Given} & \text{Goal} \\ \forall a_1 \in B \forall a_2 \in B((f(a_1) = f(a_2) \rightarrow a_1 = a_2) & a_1 = a_2 \\ \forall b_1 \in B \forall b_2 \in B((g(b_1) = g(b_2) \rightarrow b_1 = b_2) & \\ a_1 \in B, a_2 \in B, g \circ f(a_1) = g \circ f(a_2) & \end{array}$

• $g(b_1) = g(b_2)$ for $b_1 = f(a_1), b_2 = f(a_2)$. Thus, $b_1 = b_2$ and hence $a_1 = a_2$.

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Definition

The function that is one-to-one and onto are called *bijections* or *one-to-one correspondence*.

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Definition

The function that is one-to-one and onto are called *bijections* or *one-to-one correspondence*.

Theorem (5.3.1)

 $f : A \rightarrow B$. If f is one-to-one and onto, then $f^{-1} : B \rightarrow A$ is also a one-to-one and onto function.

Proof.

We show first that f⁻¹ is a function. That is ∀b∃!a ∈ A((b, a) ∈ f⁻¹). This is divided into ∀b ∈ B∃a ∈ A((b, a) ∈ f⁻¹) and

 $\forall b \in B \forall a_1 \in A \forall a_2 \in A(((b, a_1) \in f^{-1} \land (b, a_2) \in f^{-1}) \rightarrow a_1 = a_2).$

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• The first goal is equivalent to $\forall b \exists a \in A(b = f(a))$. Thus, *f* being onto implies this.

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 $\forall b \in B \forall a_1 \in A \forall a_2 \in A(((b,a_1) \in f^{-1} \land (b,a_2) \in f^{-1}) \rightarrow a_1 = a_2).$

- The first goal is equivalent to $\forall b \exists a \in A(b = f(a))$. Thus, *f* being onto implies this.
- The second goal is equivalent to

$$\forall b \in B \forall a_1 \in A \forall a_2 \in A((b = f(a_1) \land b = f(a_2)) \rightarrow a_1 = a_2).$$

Hence, f being one-to-one implies this.

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Theorem (5.3.2)

 $f : A \to B$ and $f^{-1} : B \to A$ are functions. Then $f^{-1} \circ f = i_A$ and $f \circ f^{-1} = i_B$.

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Theorem (5.3.3)

 $f: A \rightarrow B.$

- If there is a function $g: B \to A$ such that $g \circ f = i_A$, then f is one-to-one.
- If there is a function $g: B \to A$ such that $f \circ g = i_B$, then f is onto.

• 1) We are given $f : A \to B, g : B \to A$ with $g \circ f = i_A$. Our goal is $\forall a_1 \in A \forall a_2 \in A((f(a_1) = f(a_2) \to a_1 = a_2))$. Suppose $a_1, a_2 \in A$ be arbitrary and $f(a_1) = f(a_2)$. Then $a_1 = i_A(a_1) = g \circ f(a_1) = g \circ f(a_2) = i_A(a_2) = a_2$.

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- 2) We are given $f \circ g = i_B$. We show $\forall b \exists a(b = f(a))$. Let $b \in B$ be arbitrary. Then we try to guess a. We have $b = f \circ g(b) = f(g(b))$. Let a = g(b). Then b = f(a) = f(g(b)) = b.

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The following statements are equivalent:

- f is one-to-one and onto
- $f^{-1}: B \rightarrow A$ is a function.
- There is a function $g: B \to A$ such that $g \circ f = i_A$ and $f \circ g = i_B$.

Proof.

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• 1 \rightarrow 2: Theorem 5.3.1.

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- 1 \rightarrow 2: Theorem 5.3.1.
- $\bullet~2 \rightarrow 3$ Theorem 5.3.2.

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Proof.

- 1 \rightarrow 2: Theorem 5.3.1.
- $\bullet~2 \rightarrow 3$ Theorem 5.3.2.
- $\bullet~3 \rightarrow 1$ Theorem 5.3.3.

Suppose that we have $f : A \to B$ and $g : B \to A$ and $g \circ f = i_A$ and $f \circ g = i_B$. Then $g = f^{-1}$.

Proof.

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Suppose that we have $f : A \to B$ and $g : B \to A$ and $g \circ f = i_A$ and $f \circ g = i_B$. Then $g = f^{-1}$.

Proof.

• By Theorem 5.3.4 (2 \leftrightarrow 3), f^{-1} : $B \rightarrow A$ is a function.

Suppose that we have $f : A \to B$ and $g : B \to A$ and $g \circ f = i_A$ and $f \circ g = i_B$. Then $g = f^{-1}$.

Proof.

- By Theorem 5.3.4 (2 \leftrightarrow 3), f^{-1} : $B \rightarrow A$ is a function.
- By Theorem 5.3.2, $f^{-1} \circ f = i_A$.

Suppose that we have $f : A \to B$ and $g : B \to A$ and $g \circ f = i_A$ and $f \circ g = i_B$. Then $g = f^{-1}$.

Proof.

- By Theorem 5.3.4 (2 \leftrightarrow 3), f^{-1} : $B \rightarrow A$ is a function.
- By Theorem 5.3.2, $f^{-1} \circ f = i_A$.

• Then
$$g = i_A \circ g = (f^{-1} \circ f) \circ g = f^{-1} \circ (f \circ g) = f^{-1} \circ i_B = f^{-1}$$
.

• Given $f : A \rightarrow B$. $X \subset A$.

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- Given $f : A \rightarrow B$. $X \subset A$.
- The image of *X*:

$$f(X) = \{f(x) | x \in X\} = \{b \in B | \exists x \in X(f(x) = b)\}.$$

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• Inverse image of *Y* for $Y \subset B$:

$$f^{-1}(Y) = \{a \in A | f(a) \in Y\}.$$

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• Inverse image of Y for $Y \subset B$:

$$f^{-1}(Y) = \{a \in A | f(a) \in Y\}.$$

Theorem (5.4.2)

 $f : A \rightarrow B, W, X \subset A$. Then (1) $f(W \cap X) \subset f(W) \cap f(X)$. And (2) $f(W \cap X) = f(W) \cap f(X)$ if f is one to one.

• (1)

Given Goal $f: A \to B, W, X \subset A$ $f(W \cap X) \subset f(W) \cap f(X)$

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• (1) Given Goal $f: A \rightarrow B, W, X \subset A$ $f(W \cap X) \subset f(W) \cap f(X)$ Given Goal $f: A \rightarrow B, W, X \subset A$ $b \in f(W) \cap f(X)$ $b \in f(W \cap X)$ $b = f(a), a \in W \cap X$

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• (2)

Given $f: A \rightarrow B, W, X \subset A$ $\forall a_1 \in A, \forall a_2 \in A$ $((f(a_1) = f(a_2)) \rightarrow a_1 = a_2)$

 $\begin{array}{c} \text{Goal} \\ \forall b \in B \quad b \in f(W \cap X) \leftrightarrow \\ b \in f(W) \cap f(X) \end{array}$

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• (2)

Given $f: A \rightarrow B, W, X \subset A$ $\forall a_1 \in A, \forall a_2 \in A$ $((f(a_1) = f(a_2)) \rightarrow a_1 = a_2$ • \rightarrow part is done in (1)

 $\begin{array}{c} \text{Goal} \\ \forall b \in B \quad b \in f(W \cap X) \leftrightarrow \\ b \in f(W) \cap f(X) \end{array}$

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• (2)

$$egin{aligned} \mathsf{Given}\ f: \mathcal{A} &
ightarrow \mathcal{B}, \mathcal{W}, \mathcal{X} \subset \mathcal{A}\ &orall a_1 \in \mathcal{A}, orall a_2 \in \mathcal{A}\ ((f(a_1) = f(a_2))
ightarrow a_1 = a_2 \end{aligned}$$

•
$$\rightarrow$$
 part is done in (1)

• (2)(ii)

$$\begin{array}{rl} \text{Given} & \text{Goal} \\ f: A \to B, W, X \subset A & b \in f(W \cap X) \\ (\forall a_1 \in A, \forall a_2 \in A \\ ((f(a_1) = f(a_2)) \to a_1 = a_2) \\ b \in f(W) \cap f(X), \\ b = f(w) = f(x), w \in W, x \in X, w = x \end{array}$$

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Project homework

• See page 258,259 1-6. Do this individually (see klms for details)

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Project homework

- See page 258,259 1-6. Do this individually (see klms for details)
- Due date is: November 30th (Friday)

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