Logic and the set theory Lecture 15: Relations in How to Prove It.

S. Choi

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Fall semester, 2012

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• Ordered pairs and Cartesian products

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- Relations

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- More about relations

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Image: A matrix

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- Course homepages: http://mathsci.kaist.ac.kr/~schoi/logic.html and the moodle page http://moodle.kaist.ac.kr
- Grading and so on in the moodle. Ask questions in moodle.

Some helpful references

• Sets, Logic and Categories, Peter J. Cameron, Springer. Read Chapters 3,4,5.

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- http://plato.stanford.edu/contents.html has much resource.

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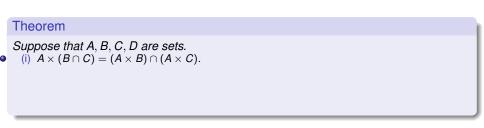
Some helpful references

- Sets, Logic and Categories, Peter J. Cameron, Springer. Read Chapters 3,4,5.
- http://plato.stanford.edu/contents.html has much resource.
- Introduction to set theory, Hrbacek and Jech, CRC Press. (Chapter 2)

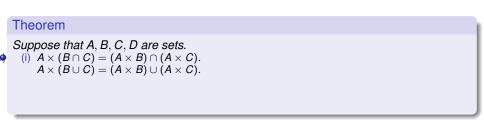
Cartesian products

- A, B sets. $A \times B = \{(a, b) | a \in A \land b \in B\}.$
- $\mathbb{R} \times \mathbb{R}$ Cartesian plane (Introduced by Descartes,.. used by Newton) First algebraic interpretation of curves...
- P(x, y) The truth set of $P(x, y) = \{(a, b) \in A \times B | P(a, b)\}.$
- x + y = 1: { $(a, b) \in \mathbb{R} \times \mathbb{R} | a + b = 1$ }.

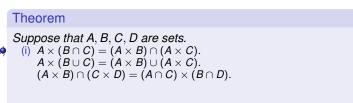
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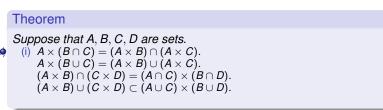


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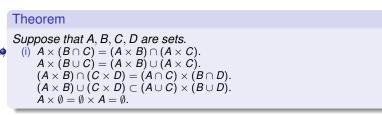


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$$\begin{array}{ll} \text{Given} & \text{Goal} \\ A,B,C & A \times (B \cap C) \subset (A \times B) \cap (A \times C) \\ & (A \times B) \cap (A \times C) \subset A \times (B \cap C) \end{array}$$

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- $R^{-1} := \{(b, a) \in B \times A | (a, b) \in R\}.$
- If S is a relation from B to another set C, then $S \circ R := \{(a, c) \in A \times C | \exists b \in B((a, b) \in R \land (b, c) \in S)\}.$

• $E = \{(c, s) \in C \times S | \text{ The student s is enrolled in course c } \}.$

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- = { $(r, c) \in R \times C$ | The student s lives in a room r and enrolled in course c.}.
- = { $(r, c) \in R \times C$ | Some student living in a room r is enrolled in course c.}.

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Theorem

- $(R^{-1})^{-1} = R.$
- $Dom(R^{-1}) = Ran(R)$.
- $Ran(R^{-1}) = Dom(R)$.
- $T \circ (S \circ R) = (T \circ S) \circ R.$
- $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$. (note order)

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Given	Goal
<i>R</i> , <i>S</i>	$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Relations

Proof of 5

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Given Goal $R, S \quad (S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Given Goal

$$R, S$$
 $(S \circ R)^{-1} \subset R^{-1} \circ S^{-1}$
 $R^{-1} \circ S^{-1} \subset (S \circ R)^{-1}$

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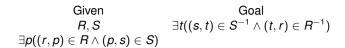
$$\begin{array}{cc} \text{Given} & \text{Goal} \\ R,S & \exists t((s,t) \in S^{-1} \land (t,r) \in R^{-1}) \\ (r,s) \in (S \circ R) \end{array}$$

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Proof of 5 continued

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$$\begin{array}{ccc} \text{Given} & \text{Goal} \\ R,S & \exists t((s,t) \in S^{-1} \land (t,r) \in R^{-1}) \\ \exists p((r,p) \in R \land (p,s) \in S) & \\ & \text{Given} & \text{Goal} \\ R,S & \exists t((r,t) \in R \land (t,s) \in S) \\ ((r,p_0) \in R \land (s,p_0) \in S) & \end{array}$$

S. Choi (KAIST)

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Theorem

• $R \subset A \times A$ is reflexive iff $i_A \subset R$.

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- 2 *R* is symmetric iff $R^{-1} = R$.
- **③** *R* is transitive iff $R \circ R \subset R$.

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