1 Introduction

About this course

- I hope that this course to be a practical one where you learn to read and write proofs yourselves. I will not present too much technical materials.
- The lecture pdf will be posted in the following pages 2-3 days before lecture:
- Course homepages: `http://mathsci.kaist.ac.kr/~schoi/logic.html` and the moodle page `http://moodle.kaist.ac.kr`
- Grading and so on in the moodle. Ask questions in moodle.
- Introduction
- Section 1: Logic. The basic purpose is to learn some elementary logic.
  - Arguments
  - Propositional logic
  - Propositional calculus
  - Predicate logic
  - Predicate calculus
- Section 2. How to prove it. We will learn how to prove mathematical statements.
  - Proofs: Proof techniques.
  - Sets, Relations, Functions
- Section 3: Naive Set theory: the basic operations of the set theory. Union, intersection...
- Section 4: Presentations: Peano axioms and arithmetics, The axiom of choice, Zorn's lemma, Well-ordering, Transfinite recursion Ordinal numbers, cardinal numbers, ordinal cardinal arithmetics.

Some helpful references

- Karel Hrbacek and Thomas Jech, Introduction to Set Theory, Pure and applied mathematics
- Joel W. Robbin, Mathematical Logic,
- Set Theory by Thomas Jech,
- Sets, Logic and Categories by Peter J. Cameron, Springer
- Changsun Choi, Introduction to Set Theory, Kyungmoon sa.(In Korean)
- `http://plato.stanford.edu/contents.html` has much resource.

2 Section 0: Our motivation

2.1 Beginning of logic

Logic

- The first people to consider logic were Greeks. Perhaps they obtained the ideas from Egypt and Babylonia which started at least 2000 years before the greek civilizations. There is no evidence that Asians studied logic.
- Logic concerns how to argue in a "sound manner".
- Socratic method: This is still the most powerful method of analysing arguments. and these are very useful in the field of law. (See `http://en.wikipedia.org/wiki/Socratic_method`)
- On the other hand, there were Sophists. They believe that truth is not knowable in some extent, and perhaps contrary to Socrates. (`http://en.wikipedia.org/wiki/Sophistry`)

Logic

- Basically Socrates, Plato, and his followers believed that by extremely careful analysis and cross-examination can lead to knowledge by building one on top of the already carefully built ideas.
- Aristotelian Syllogism: (`http://en.wikipedia.org/wiki/Syllogism`) (superceded by Frege and Russel.)
- Do we believe in progress or not? Is it all illusion?

- There are a lot of controversies here. These form the main subject of philosophy.
- The main problem here is that to understand anything, we need much interpretations. How does such happen? The things are not often what it is considered to be true.
- What we consider as reality is some rough picture that we inherit from our teachers and other people. only the conventional thinking.
- Ultimately, I think all science and mathematics belong to humanistic studies and are influenced by humanistic approaches.
- Logic as developed by Russel and so on also have much controversies where philosophers are still working on. `http://plato.stanford.edu/entries/logical-atomism/`

- On the other hand, I am also very concerned about developments such as postmordernism. One should not believe these hazy approach in the humanities too much...
- Postmordernism is quite popular. Postmordernism tries to go beyond.. to haziness?
- In general asian philosophy Confucianism, Taoism do not study logic or arguments. Neo-confucianism studied some of these questions in 16th century.
- Buddhism does but too difficult to be written down?
- What is great about western thinking was that eventhough the world is chaotic and variable, they believe that there are central laws discoverable and understandable by men.
- This is a very good principle that applies today. One should never give up hope in this regard.
- Perhaps, asian culture never tried this because of the religion? Neo-confucian scholars claim that truth is not knowable by mankind.
- If we believe in mysteries and ambiguities, then where do we end up? The systematic study and axiomatization and reductionism are all important tools to be used throughly before attempting other methods.
- These methods led us far but many western people are suggesting more holistic view.... How much will it succeed?

Mathematics and logic

- During the development of calculus, there were many controversies. Newton's infinitesimal was considered a nonsense by the philosopher Berkeley.
- Euler wrote many integral and series formula which would be considered nonsense today.
- Cauchy attacked Euler and started new criterion of limits.
- The real number system was not defined until Dedekind.
- Riemann started to define surfaces and manifolds. He assumed existence of some harmonic functions using Dirichlet principle. These were found to be groundless by Weierstrauss: The existence is equal to construction. `http://www.schillerinstitute.org/fid_02-06/2004/044_riemann_dirichlet.html`
- Cantor defined ordinals and cardinals...(Hilbert: paradise?)
- To settle these, some philosophers developed logical foundation of mathematics.

Mathematics and computer science, science, engineering, so on

- The computer science is most similar to mathematics, more to Intuitive mathematics.... It is unclear whether computer can study all of mathematics... Many mathematician think otherwise.
- Brouwer introduced Intuitive mathematics. This created a lot of fight.
- Today, the set theoretic foundation of Zermelo-Frankel that we use is essentially logic and is endorsed by Gödel.
- Today the alternative to the set theory is the category theory. The category theory can also handle intuitive mathematics. But this is much harder to learn for beginning students.
- Much of Science do not rely on logic or the set theory. This makes mathematics very distinctive.
- Often we need to round off numbers and use approximations. Computers round off numbers.
- Numerical mathematics is not mathematics.
- What is the best way for mathematics and science and engineering to communicate safely with one another? No one knows...
- Maybe these are young people's tasks. These are completely open. These are really serious questions.
- The some good solutions or ideas to above will have huge impact on a lot of things... Like artificial intelligence, language, human robot interactions, and so on.

A critique of the set theory mathematics

- One obtains answers but the answers are in the set theory.
- The set theory is not the real world...
- So how to interpret the result?
- In general modelling real world mathematically involves a lot of interpretations.... some of which are vague and furthermore, the set theory do not help you.
- The Peano-Axiom of the set of natural numbers N leads to Gödel's incompleteness. Hence the set theory is insufficient to formalize N completely.
- It is well-known that mathematical theories cannot fully justify many accepted results in physics and engineering: Feynman integral, phase change, solid state physics,... There are actually too many significant accumulations.
- In fact, because of the rigid mathematical foundation, it is very difficult to communicate with other fields. Also, many mathematicians are single-minded.

Alternative foundation

- Intuitive mathematics: Brouwer -> Heyting -> Constructive Analysis of Bishop `http://en.wikipedia.org/wiki/Intuitionism` This theory has much in common with Computer Science. `http://en.wikipedia.org/wiki/Constructive_analysis` In the end, the results are similar and the set theory settles more...
- Weyl's foundation with integers Z given.
- Category theory: currently many mathematicians are adopting it. More powerful but less easy to teach.
- New Foundation by W. V. Quinne.

My thoughts on changing foundation

- It might be a very popular idea. But the main point is that most intuitive type mathematics produce very similar results to current mathematics.
- Even if the foundation changes, most of important part of the mathematical theorems would change little.
- Something close to the set theory and encompassing many ideas of the set theory will replace our current foundation.
- Also, a different foundation is already covered by Category theory and studied.
- However, for applied mathematics much looser standard can be used very well and soundly.
- In the future, we might have a different foundation which makes the interaction with other field easier.
- Hence, one should be open minded but learn the traditional theory first.

My thought on the set theory foundation and mathematics

- The set theory was introduced by logician to settle many differences of opinions among mathematicians.
- The set theory shows us that there are no self–contradiction to theory once the set theoretical model can be built. However, the set theory cannot show that itself is without self-contradtions.
- This is a very stable system without giving us much troubles.
- With the set theory, mathematics is a very stable field and in principle without much disagreements.
- However, the set theory makes mathematics into something of an abstract theory.

- The set theory often gives you existence of objects but not how to find it.
- The set theory also makes much of mathematics very convoluted. It does hide many big problems also.