

(ALEXANDER STOJINENOW)

1. Bennequin's inequality:

Knot $S^1 \hookrightarrow S^3$



links $S^1 \cup \dots \cup S^1 \hookrightarrow S^3$



$c(D)$ # crossing



$+ + - = 0$

$w(\) = 0$

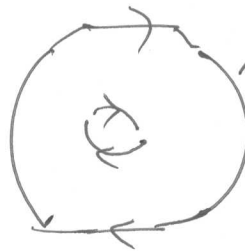
$w(D)$ = sum of signs of crossing

$S(D)$ = # of Seifert circles

E.g.



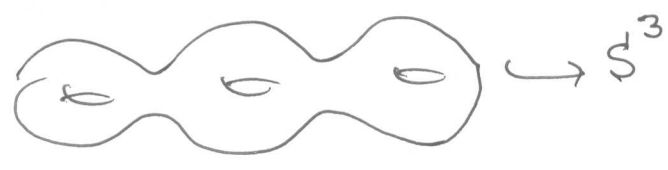
$S(\downarrow) = 3$



← Seifert circle

$S(\)$

Genus: (Seifert) $K \subset S^3 \exists S$



$\partial S = K$ Seifert Surface.

$g(S) = \text{genus}(S)$

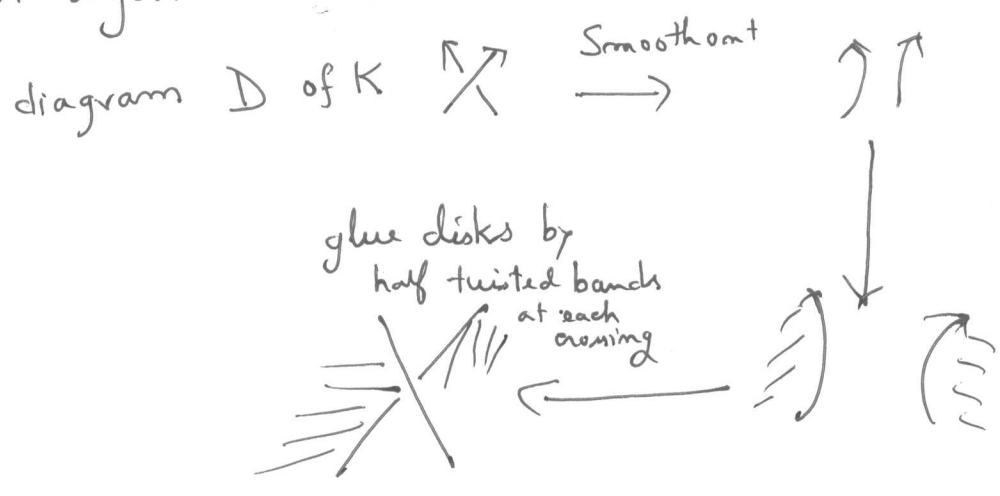
$g(K) = \min \{ g(S) : S \text{ Seifert Surface for } K \}$

genus of K

$g(K) = 0 \Leftrightarrow$



Seifert algorithm



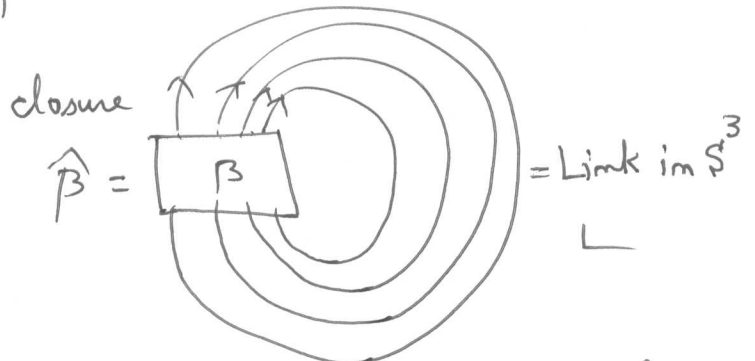
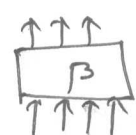
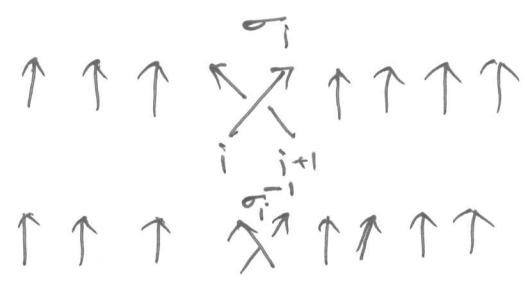
Canonical Seifert Surface of D

Braid representation:

$$B_m = \langle \sigma_1, \sigma_2, \dots, \sigma_{r-1} \mid \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \rangle$$

$1 \leq i \leq r-1$

Braid group



β is a braid representation for $L = \widehat{\beta}$.

$$B_m / [B_m, B_m] = \mathbb{Z} \text{ abelianization}$$

$$B_m \rightarrow \mathbb{Z}$$

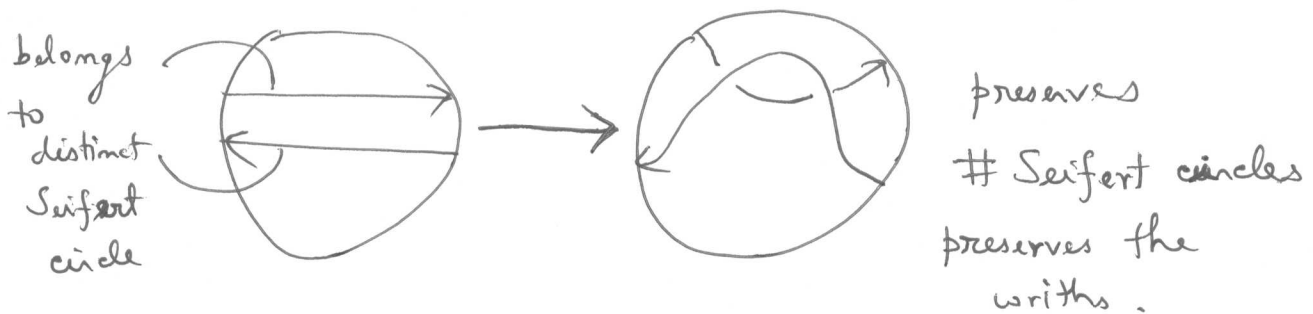
$$\sigma_i \rightarrow 1 \quad \text{exponent sum}$$

Theorem: (Bennequin '83): $K = \widehat{\beta}$ then $\beta \in B_m$
 $m = m(\beta) = \# \text{ strands of } \beta$.

$$\exists g(K) \geq |\mathbb{F}\beta| - m(\beta) + 1.$$

extends Braid representation \rightarrow diagrams.

Yamada & Vogel: ^{Thm} One can turn every link diagram into a closed braid diagram by ④



Cor: Assume D is a diagram of K

$$2g(K) \geq |w(D)| - g(D) + 1$$

2. Link polynomials $\{\text{links in } S^3\} \rightarrow \mathbb{Z}[t^{\pm 1/2}]$
 $\mathbb{Z}[t^{\pm 1}, t^{\pm 1/2}]$

$$\Delta(\text{crossing}) \rightarrow \Delta(\text{smooth}) = (t^{1/2} - t^{-1/2}) \Delta(\text{strand})$$

Alexander polynomials.

$$t^{-1} V(\text{crossing}) = (t^{1/2} - t^{-1/2}) V(\text{smooth}) \quad \text{Jones polynomial}$$

$$\Delta(0) = V(0) = 1.$$

$$t^{-1} P(\text{crossing}) - t P(\text{smooth}) = -m P(\text{strand})$$

$$P(K) = \mathbb{Z}[t^{1/2}, t^{-1/2}] \quad \text{Hom } F(Y(-PT)) \quad \text{Skein polynomial.}$$

Theorem: $g(K) \geq \frac{1}{2} \text{span } \Delta(K).$

Ex: trefoil link K . $\Delta(K) = t - 1 + t^{-1}$.

$$\text{span}(\Delta) = 2 \Rightarrow g(K) = 1.$$

