Combinatorial Topology Quiz 1 (2017 Fall)

Question 1. *Write your name, student ID number.*

Recall the definition of **lifting correspondence** as stated below.

Definition 1. *Let $p : E \to B$ be a covering map; let $b_0 \in B$. Choose e_0 so that $p(e_0) = b_0$. Given an element $[f]$ of $\pi_1(B, b_0)$, let $\widetilde{f}$ be the lifting of f to a path in E that begins at e_0. Let $\phi([f])$ denote the end point widetilde$f(1)$. Then ϕ is well defined as a map between sets*

$$\phi : \pi_1(B, b_0) \to p^{-1}(b_0).$$

Question 2. *When can we say that lifting correspondence is surjective, or even bijective? What condition on E can guarantee it?*

Question 3. *Write down a covering map $p : \mathbb{R} \to S^1$ such that if $e_0 = 0$ and $b_0 = p(e_0)$, then $p^{-1}(b_0) = \mathbb{Z}$.*

Using this, we showed that the fundamental group of S^1 is isomorphic to the additive group of integers $(\mathbb{Z}, +)$.

Question 4. *Generalize the argument for $\pi_1(S^1)$, show that the fudamental group of the torus $T = S^1 \times S^1$ is isomorphic to the group $\mathbb{Z} \times \mathbb{Z}$.*