Combinatorial Topology Quiz 0 (2017 Fall)

Question 1. *Write your name, student ID number. Did you take MAS331: Topology? If so, in which semester did you take it?*

Question 2. *What is the product topology? If there is a infinite product of topological spaces, please mention one fact which holds with respect to the product topology but not with respect to the box topology.*

Question 3. *Let A be a subset of a topological space X, and let A' be the set of all limit points of A. Show that $\bar{A} = A \cup A'$ where $\bar{A}$ denotes the closure of A.*

Question 4. *Show that if $f : X \to Y$ is continuous, and if A is a subspace of X, then the restricted function $f|_A : A \to Y$ is continuous.*

Question 5. *Let $X = A \cup B$, where A and B are closed in X. Let $f : A \to Y$ and $g : B \to Y$ be continuous. If $f(x) = g(x)$ for every $x \in A \cap B$, then f and g combine to give a new function $h : X \to Y$, defined by setting $h(x) = f(x)$ if $x \in A$, and $h(x) = g(x)$ if $x \in B$. Show that h is continuous.*

Question 6. *Consider the map $f : [0, 1) \to \mathbb{R}^2$ defined by $f(t) = (\cos 2\pi t, \sin 2\pi t)$. Is f a topological imbedding? Why?*

Question 7. *Let A be a connected subspace of X. If $A \subset B \subset \bar{A}$, show that B is also connected.*

Question 8. *Show that the components (or connected components) of a topological space X are connected disjoint subspaces of X whose union is X.*

Question 9. *What does it mean by that a collection $\mathcal{C}$ of subsets of X has the finite intersection property? Use this property to give an equivalent definition of the compactness of a space.*