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ERRATUM: AN ANALYSIS OF A BROKEN P_1-NONCONFORMING FINITE ELEMENT METHOD FOR INTERFACE PROBLEMS*

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Abstract. The object of this note is to correct an error in the proof of Theorem 3.4 of the paper [*An analysis of a broken P_1-nonconforming finite element method for interface problems*, SIAM J. Numer. Anal., 48 (2010), pp. 2117–2134]. As a result, Theorem 3.4 requires a higher regularity than the usual elliptic interface problems can have, i.e., $\beta\nabla p \in \left(H^{1/2+\epsilon}(\Omega)\right)^2$ $(0 < \epsilon < 1/2)$. Hence we point out that even though the result now holds under this extra regularity assumption, the regularity is unlikely to hold for general interface problems. Thus the result has some limited usage.

Key words. P_1-nonconforming immersed finite element, interface problems, regularity assumptions, correction, fractional order trace estimate

AMS subject classifications. 65N15, 65N30, 35J60

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1. Introduction. In the paper [1], the authors proposed an immersed finite element method (IFEM) based on the Crouzeix–Raviart P_1-nonconforming linear finite element. In the proof of Theorem 3.4 (in the line above (3.8)), we used the claim $\beta\frac{\partial p}{\partial n} \in H^1(T)$, which is absurd. We briefly give a new proof below. To prove the theorem, we need a higher regularity than the usual one. In this regard, we remove Remark 3.1. Finally, we fix a typographical error in Lemma 2.4. The equation, lemma, and theorem numbers here refer to those of [1].

2. Correction. In this section, we restate Theorem 3.4 with an extra regularity assumption and prove it.

Theorem 3.4. *Let $p \in \widetilde{H}^2(\Omega)$ be the solution of the problem (2.3) and $p_h \in \widehat{S}_h(\Omega)$ be the corresponding IFEM solution. Assume that $\beta\nabla p \in \left(H^{1/2+\epsilon}(\Omega)\right)^2$ for some $0 < \epsilon < 1/2$ holds. Then there exists a constant $C > 0$ such that*

$$\|p - p_h\|_0 + h^{1/2+\epsilon}\|p - p_h\|_{1,h} \le Ch^{1+2\epsilon}\|f\|_{0,\Omega}.$$

Proof. Consider the H^1-error term only. The L^2-error would follow similarly by a duality argument. We first prove the case when $\epsilon = 1/2$. By the second Strang lemma, it suffices to estimate the consistency term in (3.7) of [1]. For each edge e of T, fix a normal vector $\mathbf{n}_e$, which we regard as defined on all of T. Consider the function defined by $\psi(x, y) = \beta\nabla p(x, y) \cdot \mathbf{n}_e$. Then $\psi(x, y) \in H^1(T)$, and we see by

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Lemma 3.3 that

$$\begin{aligned}\left|\sum_{T\in\mathcal{T}_h} < \beta\frac{\partial p}{\partial n}_e, \phi_h >_{\partial T}\right| &= \left|\sum_{T\in\mathcal{T}_h}\sum_{e\subset\partial T} < \beta\frac{\partial p}{\partial n_e} - \overline{\beta\frac{\partial p}{\partial n}_e}, \phi_h >_e\right| \\ &= \left|\sum_{T\in\mathcal{T}_h}\sum_{e\subset\partial T}\int_e(\psi-\overline{\psi})\phi_h ds\right| \\ &\leq Ch\sum_{T\in\mathcal{T}_h}\|\psi\|_{1,T}|\phi_h|_{1,T} \leq Ch\|p\|_{\tilde{H}^2(\Omega)}|\phi_h|_{1,h}.\end{aligned}$$

Hence we get the result when $\epsilon = 1/2$. The general case follows by applying the fractional order trace estimate and the fractional order Deny–Lions lemma [2], [3]. ◻

Finally, we correct a typographical error in the proof of Lemma 2.4: In line 3 of the proof, "$\widehat{S}_h(T)$" must be "p belongs to $\widehat{S}_h(T)$."

Remark 3.1. We note that the regularity $\beta\nabla p \in \left(H^{1/2+\epsilon}(\Omega)\right)^2$ is unlikely to hold for general interface problems. Hence this proof of Theorem 3.4 is limited to certain special problems having the regularity. In the future, we will investigate more on this topic.

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