Generalized OFDM for 5th Generation Mobile Communications

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***Abstract*— In this paper, we propose a generalized OFDM (G-OFDM) to map data vectors to channel vectors through filter matrices unlike the conventional OFDM mapping data to subchannels by one to one. Filter matrices play a role to control the spectrums of input data vectors to reduce spectral leakages significantly unlike OFDM which does not have controllability for spectrum. The filter matrix consists mainly of an initial matrix, a jump matrix and the nearest matrix with orthonormal columns. The initial matrix is constructed with minimum nonzero elements to minimize operations when multiplying with other matrix. The jump matrix is constructed by a proper matrix from the initial matrix so that the sum of all elements for each column vector of the matrix becomes zero after subtraction. In this method, a pilot vector to estimate a communication channel which is very similar to pilot symbol inserting methods in OFDM is included in the filter matrix. It was confirmed that the proposed G-OFDM has a superior characteristic in reducing spectral leakage to that of OFDM.**

Keywords— Filter; matrix; G-OFDM; multiplexing; OFDM; orthogonal.

I. Introduction

OFDM plays a significant role in modern telecommunications, ranging from its use in DSL-modem technology to 802.11 Wi-Fi wireless systems [1]. Recently, many efforts have been focused on studies on new transmission technologies to enhance the spectral characteristic of OFDM for the next generation mobile communications. FBMC (filter bank multi-carrier) and UFMC (universal filtered multi-carrier) to reduce the spectral leakage of OFDM are prominent outcomes [2-3]. FBMC has the merit of being able to satisfactorily to reduce the spectral leakage to neighbor channels by filtering in the frequency domain, but two or four sampling per symbol may be a drawback against to high-speed transmission since the packet lengths become longer actually. UFMC is a scheme to mitigate the spectral leakage by filtering using general filters, but the filtering in the time domain causes excessive computations due to performing convolution and it also lengthens the packet due to filter response.

We introduce G-OFDM which reduces the spectral spread occurred in the conventional OFDM effectively without changing the structure of OFDM. Since G-OFDM performs filtering in the frequency domain with reasonable operational complexity and it does not change the length of the OFDM packet, it can accommodate the existing OFDM based systems.

II. G-OFDM

A data vector $\boldsymbol{d}$ is divided into $\boldsymbol{J}$ small vectors, the i-th data vector $\boldsymbol{d}_i$ is multiplied with the filter matrix $\boldsymbol{G}$, and each product becomes the i-th filtered vector $\boldsymbol{v}_i$. A data symbol in a data vector is carried on the i-th column vector in the filter matrix $\boldsymbol{G}$. Hence, column vectors of the matrix $\boldsymbol{G}$ should be orthogonal in order not to interfere each other between the symbol data and they should reduce the spectrum leakage without any change in the structure of the conventional OFDM. Vectors $\boldsymbol{v}_i$'s are merged into one vector $\boldsymbol{v}$, transformed by the IFFT, and they are transmitted in the form of the vector $\boldsymbol{s}$. The relation for orthogonality to recover the data symbol without interference between symbols is that $\boldsymbol{G}^*\boldsymbol{G} = \boldsymbol{I}$.

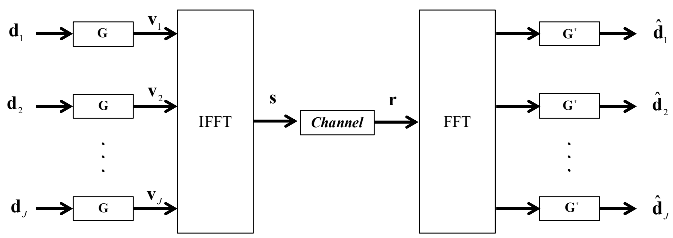


Fig. 1. G-OFDM

III. Filter Matrix

A. Filter Matrix Generation

A filter matrix $\boldsymbol{G}$ is generated with the initial matrix $\boldsymbol{G}_0$ according to the signal flow diagram shown in Fig.2. From the figure, we can obtain a filter matrix from the matrix function

$$\boldsymbol{G} = f\left(\boldsymbol{\Omega}\boldsymbol{\Theta}\boldsymbol{G}_0\right) \tag{1}$$

where

$$\boldsymbol{\Theta} = \boldsymbol{W}^T \boldsymbol{F} \boldsymbol{\Phi} \boldsymbol{F}^{-1} \boldsymbol{W},$$

$$\boldsymbol{\Omega} = \boldsymbol{W}^T \boldsymbol{F} \boldsymbol{\Psi} \boldsymbol{F}^{-1} \boldsymbol{W},$$

$$f(\boldsymbol{X}) = \boldsymbol{X}\left(\boldsymbol{X}^* \boldsymbol{X}\right)^{-\frac{1}{2}}.$$

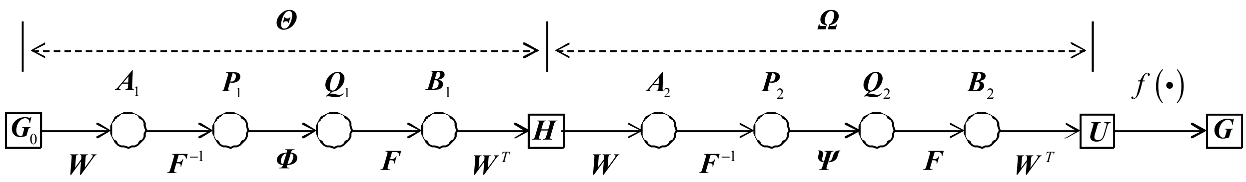


Fig. 2. Signal flow diagram for filter matrix generation. $\boldsymbol{\Theta}$ is a jump removing matrix and $\boldsymbol{\Omega}$ is a filtering matrix .

B. Initial Matrix

We define an initial filter $N \times (N-1)$ matrix by

$$\boldsymbol{G}_0 = \begin{bmatrix} & & & & 1 & 1 \\ & & & \iddots & & \\ & & 1 & 1 & & \\ & 1 & 1 & & & \\ 1 & & & & & \\ -1 & & & & & \\ & 1 & -1 & & & \\ & & 1 & -1 & & \\ & & & \ddots & & \\ & & & & 1 & -1 \end{bmatrix} \quad (2)$$

where N is even. This matrix is constructed so that all columns have minimum nonzero elements, having unit length and orthogonal each other. In the matrix all blanks are zeros and it is obvious that $\boldsymbol{G}_0^* \boldsymbol{G}_0 = \boldsymbol{I}$.

B. Permutation and Zero Padding

In order to process $\boldsymbol{G}_0$ in the time domain, we transform it from the frequency domain to the time domain. The initial matrix $\boldsymbol{G}_0$ is padded by zeros and permuted by $\boldsymbol{W}$ as

$$\boldsymbol{A}_1 = \boldsymbol{W} \boldsymbol{G}_0 \quad (3)$$

where $\boldsymbol{W}$ is

$$\boldsymbol{W} = \begin{bmatrix} \left.\begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \end{matrix}\right\} M \\ \left.\begin{matrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{matrix}\right\} L-M \end{bmatrix}.$$

where L and M are lengths of columns and rows respectively.

C. Inverse Discrete Fourier Transform (IDFT)

We take the IDFT of $\boldsymbol{A}_1$ to transform it to a time domain matrix as

$$\boldsymbol{P}_1 = \boldsymbol{F}^{-1} \boldsymbol{A}_1 \quad (4)$$

where $\boldsymbol{F}^{-1}$ is the inverse DFT matrix of

$$\boldsymbol{F} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & e^{-j\theta} & e^{-j2\theta} & \cdots & e^{-j(L-1)\theta} \\ 1 & e^{-j2\theta} & e^{-j4\theta} & \cdots & e^{-j2(L-1)\theta} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & e^{-j(L-1)\theta} & e^{-j2(L-1)\theta} & \cdots & e^{-j(L-1)(L-1)\theta} \end{bmatrix} \quad (5)$$

where $\theta = 2\pi / L$. Since the first row of the inverse DFT matrix is $\overbrace{[1, \cdots, 1]}^{L}$, the first row of $\boldsymbol{P}_1$ becomes

$$\boldsymbol{p}_1 = \left[\sum_{l=0}^{L-1} a_{l,1} \quad \sum_{l=0}^{L-1} a_{l,2} \quad \cdots \quad \sum_{l=0}^{L-1} a_{l,\mathrm{M}} \right]. \quad (6)$$

In OFDM, the signal spectrum spread phenomenon is caused by abrupt jumps in carrier functions in the time domain. We can remove jumps in columns of the matrix $\boldsymbol{P}_1$ by subtracting the first row from each row of $\boldsymbol{P}_1$ as

$$\boldsymbol{Q}_1 = \boldsymbol{\Phi} \boldsymbol{P}_1 = \begin{bmatrix} \boldsymbol{p}_1 \\ \vdots \\ \boldsymbol{p}_L \end{bmatrix} - \begin{bmatrix} \boldsymbol{p}_1 \\ \vdots \\ \boldsymbol{p}_1 \end{bmatrix} = \boldsymbol{P} - \boldsymbol{J}_P \quad (7)$$

where $\boldsymbol{\Phi}$ is a jump removing operator and $\boldsymbol{J}_P = \begin{bmatrix} \boldsymbol{p}_1 & \cdots & \boldsymbol{p}_1 \end{bmatrix}^T$ is a jump matrix.

D. Discrete Fourier transform

We take the DFT of $\boldsymbol{Q}_1$ to transform it to a frequency domain matrix as

$$\boldsymbol{B}_1 = \boldsymbol{F} \boldsymbol{Q}_1 \quad (8)$$

where $\boldsymbol{F}$ is the DFT matrix.

E. Permutation and Truncation

Performing permutation and truncation, we have an $M \times N$ matrix

$$\boldsymbol{H} = \boldsymbol{W}^T \boldsymbol{B}_1 \tag{9}$$

where $\boldsymbol{W}^T$ is the permuting and band-limiting $M \times L$ matrix.

F. Frequency Domain Jump Removal

Besides removing jumps represented by (7) in the time domain, we can remove jumps in the frequency domain directly. The relation between $\boldsymbol{G}_0$ and $\boldsymbol{H}$ is given by

$$\boldsymbol{H} = \boldsymbol{\Theta}\boldsymbol{G}_0. \tag{10}$$

The matrix $\boldsymbol{\Theta}$ in the right-hand term in (10) is the matrix transforming $\boldsymbol{G}_0$ to $\boldsymbol{H}$, where the sum of all elements for each column of $\boldsymbol{H}$ is zero. In other words

$$\boldsymbol{1}_{1\times N}\boldsymbol{H} = \boldsymbol{0}_{1\times(N-1)}. \tag{11}$$

There may be numerous matrices satisfying (10) in the frequency domain, but for simplicity we choose a matrix to make the sum of each column zero by subtracting a matrix. An important thing is that the rank of the jump removed matrix should not be less than that of the original matrix. We define a $N \times (N-1)$ jump matrix by

$$\boldsymbol{R} = \begin{bmatrix} & & & & & & & 0 \\ & & & & & 0 & 0 & \\ & & & & \iddots & & & \\ & & 0 & 0 & & & & \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 & 0 \\ & & 0 & 0 & & & & \\ & & & & \ddots & & & \\ & & & & & 0 & 0 & \\ & & & & & & & 0 \end{bmatrix}. \tag{12}$$

Subtracting (12) from (2), we obtain a jump removed matrix as

$$\boldsymbol{H} = \boldsymbol{\Theta}\boldsymbol{G}_0 = \boldsymbol{G}_0 - \boldsymbol{R} = \begin{bmatrix} & & & & & & 1 \\ & & & & & 1 & \\ & & & 1 & \iddots & & \\ & 1 & 1 & & & & \\ 1 & -1 & 0 & -1 & \cdots & 0 & -1 \\ -1 & -1 & 0 & -1 & \cdots & 0 & -1 \\ & 1 & -1 & & & & \\ & & & 1 & \ddots & & \\ & & & & & -1 & \\ & & & & & & 1 \end{bmatrix} \tag{13}$$

where $\boldsymbol{\Theta}$ is the jump removing operator and the rank of $\boldsymbol{H}$ is $N-1$ [4].

G. Signal Filtering

Signal filtering of the jump removed matrix is performed by

$$\boldsymbol{Q}_2 = \boldsymbol{\Psi}\boldsymbol{F}^{-1}\boldsymbol{W}\boldsymbol{H}, \tag{14}$$

where a filtering $N \times (N-1)$ matrix $\boldsymbol{\Psi}$ is given by

$$\boldsymbol{\Psi} = \frac{1}{2}\begin{bmatrix} 1 & & & & \\ 1 & 1 & & & \\ & 1 & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix}. \tag{15}$$

H. Frequency Domain Representation of the Filtered Matrix

After filtering, we apply the DFT, permutation and truncation. Thus, we obtain a filtered matrix

$$\boldsymbol{U} = \boldsymbol{W}\boldsymbol{F}\boldsymbol{Q}_2 = \boldsymbol{\Omega}\,\boldsymbol{H} = \boldsymbol{\Omega}\boldsymbol{\Theta}\boldsymbol{G}_0, \tag{16}$$

where $\boldsymbol{\Omega} = \boldsymbol{W}^T\boldsymbol{F}\boldsymbol{\Psi}\boldsymbol{F}^{-1}\boldsymbol{W}$.

I. Nearest Matrix with Orthonormal Columns

We compute the nearest matrix with orthonormal columns from the formula [5]

$$\boldsymbol{G} = \boldsymbol{U}\left(\boldsymbol{U}^*\boldsymbol{U}\right)^{-\frac{1}{2}} \tag{17}$$

where $\boldsymbol{U}^*$ is a Hermitian matrix of $\boldsymbol{U}$. There may be numerous nearest matrices depending on initial matrices. In view of orthogonality, one matrix is not necessarily superior to the other since they have all orthonormal columns and the distances between columns are equal, but it still remains some issues of spectral spread to be discussed.

J. Filter Matrix with Pilot Vector

In wireless communications, there may be many paths between a transmitter and a receiver, so we must sense a communication channel to recover the received signal correctly. In the conventional OFDM, pilot symbols are inserted regularly in the packet at the transmitter, and the receiver estimates the channel using them. In G-OFDM, pilot vectors are used without being affected by remaining vectors and without degradation of spectral characteristic. A pilot column vector with two nonzero entries which are 1 and -1 can be included in the initial filter $N \times (N-2)$ matrix. all the entries in the rows corresponding to nonzero entries of the pilot vector should be zeros. An example inserting a pilot vector in the 6th column is as follows:

$$\bar{\boldsymbol{G}}_0 = \left[\begin{array}{ccccc|c|ccccc} & & & & & & & & & 1 & 1 \\ & & & & & & & & \iddots & & \\ & & & & & & 1 & 1 & & & \\ & & & & & 1 & & & & & \\ & & & 1 & 1 & & & & & & \\ & 1 & 1 & & & & & & & & \\ 1 & & & & & & & & & & \\ -1 & & & & & & & & & & \\ & -1 & 1 & & & & & & & & \\ & & & -1 & 1 & & & & & & \\ & & & & & -1 & & & & & \\ & & & & & & -1 & 1 & & & \\ & & & & & & & & \ddots & & \\ & & & & & & & & & -1 & 1 \end{array}\right] \quad (18)$$

which is the matrix omitting the second vector in the initial filter matrix $\boldsymbol{G}_0$ and the corresponding jump $N \times (N-2)$ matrix can be defined by

$$\bar{\boldsymbol{R}} = \left[\begin{array}{ccccc|c|cccccc} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & \cdots & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \end{array}\right] \quad (19)$$

which is also the matrix obtained by omitting the column from the jump matrix $\boldsymbol{R}$. We can obtain a form similar to (10) given by

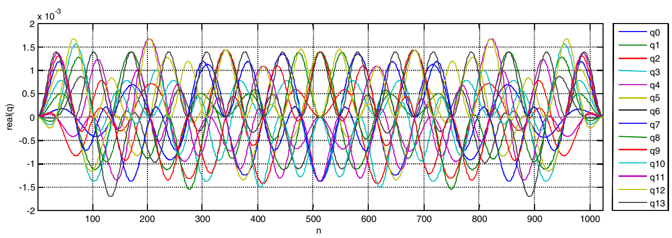
$$\bar{\boldsymbol{H}} = \boldsymbol{\Theta}\bar{\boldsymbol{G}}_0 = \bar{\boldsymbol{G}}_0 - \bar{\boldsymbol{R}}. \quad (20)$$

Lastly, replacing $\boldsymbol{\Theta G}_0$ in (1) by (20), we obtain the pilot included filter matrix [6]

$$\bar{\boldsymbol{G}} = f\left(\boldsymbol{\Omega}\left[\bar{\boldsymbol{G}}_0 - \bar{\boldsymbol{R}}\right]\right). \quad (21)$$

H. Example

With an initial matrix $\bar{\boldsymbol{G}}_0^{16\times14}$ and a jump matrix $\bar{\boldsymbol{R}}^{16\times14}$, from (20) we have $\bar{\boldsymbol{G}}^{16\times14} = \bar{\boldsymbol{G}}_{real}^{16\times14} + j\bar{\boldsymbol{G}}_{imag}^{16\times14}$ where $\bar{\boldsymbol{G}}_{real}^{16\times14}$ and $\bar{\boldsymbol{G}}_{imag}^{16\times14}$ are given in the Appendix. We can see that all elements except the 5^{th} and 12^{th} elements of the 6^{th} column of these two matrices are zeros and all elements in the 5^{th} and 12^{th} rows except the 6^{th} column are zeros. Therefore, the first column is not affected by remaining columns over the multipath fading channel, and hence the first column can be used as a pilot vector.



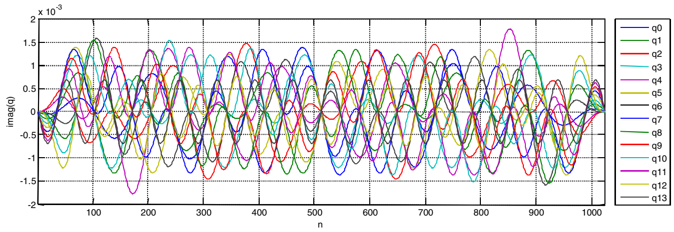


Fig. 3. Time domain responses of columns of the filter matrix $\boldsymbol{G}$.

In Fig. 3, all curves start at zero and end at zero since we removed jumps. Fig. 4 shows the frequency response of G(16,14)-OFDM. Fig. 5 shows the power spectral densities of OFDM with 14 subchannels and G(16,14)-OFDM.

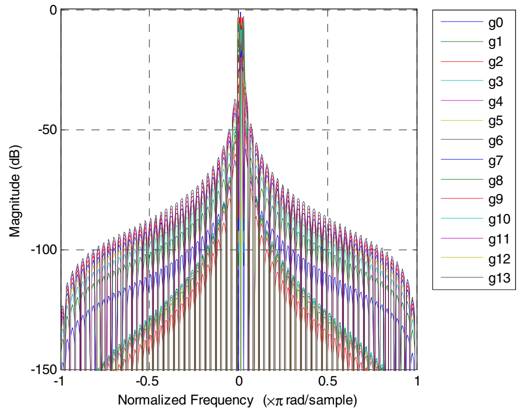


Fig. 4. Frequency response of G(16,14)-OFDM. The FFT size is L = 1024.

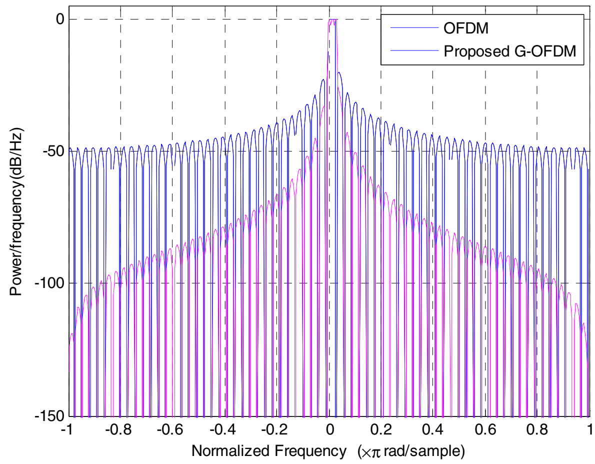


Fig. 5. The power spectral densities of OFDM with 14 subchannels and G(16,14)-OFDM. The power of G-OFDM decreases drastically according to frequency compared with OFDM.

III. Conclusion

G-OFDM which is able to reduce the spectral leakage and to implement with fewer operations than the previously developed schemes was proposed. We developed a method to generate pilot added filter matrices which are not affected by any column vectors. We showed that G-OFDM has a better spectral characteristic than that of the conventional OFDM. Since G-OFDM performs filtering in the frequency domain, it can reduce significantly the amount of operations compared with any schemes performing filtering in the time domain. Since G-OFDM does not lengthen packets of the conventional OFDM, it can support current technologies such as LTE and 802. 11 Wi-Fi systems.

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Appendix: The filter matrix $\bar{G}^{16\times14}$

$\bar{G}_{real}^{16\times14}$

0.0000	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	0.6338	0.7072
0.0000	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	0.6338	0.7071	-0.0733	0.0000
0.0000	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	0.6338	0.7071	-0.0733	0.0000	-0.0733	0.0000
0.0000	0.0000	-0.0733	0.0000	-0.0733	0.0000	0.6338	0.7071	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.7072	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0733	0.7070	0.6337	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000
0.0000	0.7070	0.6337	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000
0.7069	0.0000	-0.2672	0.0000	-0.2672	0.0000	-0.2672	0.0000	-0.2672	0.0001	-0.2672	0.0001	-0.2672	0.0001
-0.7069	0.0000	-0.2672	0.0000	-0.2672	0.0000	-0.2672	0.0000	-0.2672	0.0001	-0.2672	0.0001	-0.2672	0.0001
0.0000	-0.7068	0.6336	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000
0.0000	0.0000	-0.0733	-0.7068	0.6335	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	-0.7065	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0733	0.0000	-0.0733	0.0000	0.6334	-0.7066	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000
0.0000	0.0000	-0.0733	0.0000	-0.0733	0.0000	-0.0733	0.0000	0.6333	-0.7065	-0.0733	0.0000	-0.0733	0.0000
0.0000	0.0000	-0.0732	0.0000	-0.0732	0.0000	-0.0733	0.0000	-0.0733	0.0000	0.6332	-0.7064	-0.0733	0.0000
0.0000	0.0000	-0.0732	0.0000	-0.0732	0.0000	-0.0732	0.0000	-0.0732	0.0000	-0.0733	0.0000	0.6331	-0.7063

$\bar{G}_{imag}^{16\times14}$

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0002	0.0000	-0.0002	0.0000	-0.0002	0.0000	-0.0002	0.0000	0.0019	0.0022	-0.0002	0.0000
0.0000	0.0000	-0.0004	0.0000	-0.0004	0.0000	-0.0004	0.0000	0.0039	0.0043	-0.0004	0.0000	-0.0004	0.0000
0.0000	0.0000	-0.0007	0.0000	-0.0007	0.0000	0.0058	0.0065	-0.0007	0.0000	-0.0007	0.0000	-0.0007	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0087	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0011	0.0108	0.0097	0.0000	-0.0011	0.0000	-0.0011	0.0000	-0.0011	0.0000	-0.0011	0.0000
0.0000	0.0130	0.0117	0.0000	-0.0013	0.0000	-0.0013	0.0000	-0.0013	0.0000	-0.0013	0.0000	-0.0013	0.0000
0.0152	0.0000	-0.0057	0.0000	-0.0057	0.0000	-0.0057	0.0000	-0.0057	0.0000	-0.0057	0.0000	-0.0057	0.0000
-0.0174	0.0000	-0.0066	0.0000	-0.0066	0.0000	-0.0066	0.0000	-0.0066	0.0000	-0.0066	0.0000	-0.0066	0.0000
0.0000	-0.0195	0.0175	0.0000	-0.0020	0.0000	-0.0020	0.0000	-0.0020	0.0000	-0.0020	0.0000	-0.0020	0.0000
0.0000	0.0000	-0.0022	-0.0217	0.0194	0.0000	-0.0022	0.0000	-0.0022	0.0000	-0.0022	0.0000	-0.0022	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	-0.0239	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0027	0.0000	-0.0027	0.0000	0.0233	-0.0260	-0.0027	0.0000	-0.0027	0.0000	-0.0027	0.0000
0.0000	0.0000	-0.0029	0.0000	-0.0029	0.0000	-0.0029	0.0000	0.0253	-0.0282	-0.0029	0.0000	-0.0029	0.0000
0.0000	0.0000	-0.0031	0.0000	-0.0031	0.0000	-0.0031	0.0000	-0.0031	0.0000	0.0272	-0.0304	-0.0031	0.0000
0.0000	0.0000	-0.0034	0.0000	-0.0034	0.0000	-0.0034	0.0000	-0.0034	0.0000	-0.0034	0.0000	0.0292	-0.0325