Program and Abstracts

# 2014 KAIST-FUDAN Workshop on Analysis and Applied Mathematics

August 27-28, 2014

Sponsored by Department of Mathematical Sciences, KAIST

> Organizers: Prof. Chang-Ock Lee, KAIST Prof. Zongmin Wu, Fudan Univ.

# Workshop Schedule

August 26 (Tue) 18:00

# Welcome Reception

# August 27 (Wed) Room 3221, Industrial Engineering Bldg (E2-1)

	09:10-09:20		Opening Ceremony
Chair:	09:20-09:50	Do Y. Kwak	A modified $P_1$ - immersed finite element method
CO Lee	09:50-10:20	Yimin Wei	Statistical Condition Estimates and Randomized
			Algorithms for Large-Scale Total Least Squares
			Problems
	10:20-10:50	Pengpeng Xie	Stability of Generalized Inversion Formulas for
			Toeplitz Matrices
	10:50-11:20		Break
Chair:	11:20-11:50	Soonsik Kwon	Asymptotic behaviors of galactic dynamics
YZhou	11:50-12:20	Zhiqiang Wang	Stabilization of extrusion process modeled by
			hyperbolic systems coupled through a moving
			interface
	12:20-14:00		Lunch
Chair:	14:00-14:30	Yong Jung Kim	Starvation driven dispersal strategies of biological
Z Wu			organisms
	14:30-15:00	Hao Wu	Well-posedness and longtime behavior of a
			hydrodynamic system modeling vesicle and fluid
			interactions
	15:00-15:30	Jaeyoung Byeon	Nonlinear elliptic problems in a periodic setting
	15:30-16:00		Break

Chair:	16:00-16:30	Yi Zhou	An elementary proof of Strauss conjecture
DY Kwak	16:30-17:00	Kil H. Kwon	Recovery of missing samples in multi-channel
			oversampling
	17:00-17:30	Zongmin Wu	A Meshless Symplectic Algorithm for multi-variant
			Hamiltonian PDEs with Radial Basis
			Approximation
	17:30		Taking Photo
	18:00		Banquet

# August 28 (Thu) Room 3221, Industrial Engineering Bldg (E2-1)

Chair:	09:20-09:50	Wenwu Gao	A quasi-interpolation scheme for periodic data
KH Kwon			based on multiquadric trigonometric B-splines
	09:50-10:20	Ganguk Hwang	Optimal Throughput and High Short Term
			Fairness Design of Renewal Access Protocol
	10:20-10:50	Liqiang Lu	Efficient algorithms for computing the non and
			semi-parametric maximum likelihood estimates
			with panel count data
	10:50-11:20		Break
Chair:	10:50-11:20 11:20-11:50	Mikyoung Lim	Break Asymptotics of the solution to the conductivity
Chair: L Lu	10:50-11:20 11:20-11:50	Mikyoung Lim	Break Asymptotics of the solution to the conductivity equation in the presence of adjacent circular
Chair: L Lu	10:50-11:20 11:20-11:50	Mikyoung Lim	Break Asymptotics of the solution to the conductivity equation in the presence of adjacent circular inclusions with finite conductivities
Chair: L Lu	10:50-11:20 11:20-11:50 11:50-12:20	Mikyoung Lim Yunxin Zhang	Break Asymptotics of the solution to the conductivity equation in the presence of adjacent circular inclusions with finite conductivities Cell motility driven by motor proteins
Chair: L Lu	10:50-11:20 11:20-11:50 11:50-12:20 12:20-13:30	Mikyoung Lim Yunxin Zhang	Break Asymptotics of the solution to the conductivity equation in the presence of adjacent circular inclusions with finite conductivities Cell motility driven by motor proteins Lunch

Invited Talk: 09:20, August 27 (Wed)

### A modified $P_1$ - immersed finite element method

#### Do Y. Kwak

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#### Abstract.

In recent years, the immersed finite element methods (IFEM) introduced in [5], [6] to solve elliptic problems having an interface in the domain due to the discontinuity of coefficients are getting more attentions of researchers because of their simplicity and efficiency. Unlike the conventional finite element methods, the IFEM allows the interface cut through the interior of the element, yet after the basis functions are altered so that they satisfy the flux jump conditions, it seems to show a reasonable order of convergence.

In this paper, we propose an improved version of the  $P_1$  based IFEM by adding the line integral of flux terms on each element. This technique resembles the discontinuous Galerkin (DG) method, however, our method has much less degrees of freedom than the DG methods since we use the same number of unknowns as the conventional  $P_1$  finite element method.

We prove  $H^1$  and  $L^2$  error estimates which are optimal both in order and regularity. Numerical experiments were carried out for several examples, which show the robustness of our scheme.

### References

- D. Arnold, F. Brezzi, B. Cockburn, and D. Marini, *Discontinuous Galerkin methods for elliptic problems*, in Discontinuous Galerkin Methods. Theory, Computation and Applications, B. Cockburn, G. E. Karniadakis, and C.-W. Shu, eds., Lecture Notes in Comput. Sci. Engrg. 11, Springer-Verlag, NewYork, 2000, pp. 89–101.
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- [5] Z. Li, T. Lin and X. Wu, New Cartesian grid methods for interface problems using the finite element formulation, Numer. Math. 96 (2003), pp. 61-98.
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Invited Talk: 09:50, August 27 (Wed)

# Statistical Condition Estimates and Randomized Algorithms for Large-Scale Total Least Squares Problems

#### Yimin Wei

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#### Abstract.

In this talk, we present new perturbation analysis and randomized algorithms for the total least squares (TLS) problem. Our new perturbation results are sharper than the earlier ones.

# Stability of Generalized Inversion Formulas for Toeplitz Matrices

#### Pengpeng Xie<sup>1</sup>

School of Mathematical Sciences, Fudan University

#### Abstract.

In this talk, we give the stability analysis on the Gohberg-Semencul-like formula regarding the Moore-Penrose inverses of Toeplitz matrices. Moreover, we develop an algorithm for the computation of Moore-Penrose inverses based on this formula and the method LSQR. The performance of this algorithm can be shown to beat the Newton's iteration by numerical results. (Joint with Yimin Wei)

 $<sup>^{1}{\</sup>rm Graduate\ student}$ 

Invited Talk: 11:20, August 27 (Wed)

# Asymptotic behaviors of galactic dynamics

Soonsik Kwon

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**Abstract**. The classical model that describes galaxy formation is the three dimensional gravitational Vlasov-Poisson systems. The system enjoys rich dynamical consequences due to the competition between free streaming in phase space and self-reinforcing effect by gravitation. Most interesting objects are spherically symmetric steady state solutions that represent static galaxies. We first review recent progress in the stability theory of spherically symmetric steady state solutions. On the other hands, when solutions disperse out we expect the asymptotic completeness. In fact, we can show a modified version of scattering. This is a joint work with Sun-Ho Choi.

Invited Talk: 11:50, August 27 (Wed)

# Stabilization of extrusion process modeled by hyperbolic systems coupled through a moving interface

#### Zhiqiang Wang

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#### Abstract.

In this talk, we consider a physical model of the extrusion process, which is described by two systems of conservation laws coupled through a moving interface. We first study the well-posedness of both the open-loop and closed-loop system. Then using a Lyapunov approach, we obtain the exponential stabilization for the closed-loop system under natural feedbacks.

### Starvation driven dispersal strategies of biological organisms

Yong Jung Kim Department of Mathematical Sciences, KAIST yongkim@kaist.edu

#### Abstract.

Non-uniform dispersal appears in many places and this talk is about such an example from mathematical ecology models. The main purpose of this talk is to introduce a diffusion model for biological organisms that increase their motility when food is insufficient, which is called *starvation driven dispersal*. It will be discussed in this talk that Fick's diffusion law does not explain such a starvation driven diffusion correctly. The diffusion model for nonuniform Brownian motion is introduced in this talk and a Fokker–Planck type diffusion law is obtained. Lotka–Volterra type competition systems with spatial heterogeneity are tested, where one species follows the starvation driven diffusion and the other follows the linear diffusion. In heterogeneous environments the starvation driven diffusion turns out to be a better survival strategy than the linear one. Various issues such as the global asymptotic stability, convergence to an ideal free distribution, the extinction and coexistence of competing species are discussed. Various phenomena can be explained using non-uniform dispersal including chemotaxis, thermal diffusion, Ludwig-Soret effect and much more.

### References

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- [5] Sun-Ho Choi and Y.-J. Kim, Chemotactic traveling waves by metric of food, preprint.
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Invited Talk: 14:30, August 27 (Wed)

# Well-posedness and longtime behavior of a hydrodynamic system modeling vesicle and fluid interactions

#### Hao Wu

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#### Abstract.

In this talk, we discuss a hydrodynamic system modeling the deformation of vesicle membranes in incompressible viscous fluids. The system consists of the Navier-Stokes equations coupled with a fourth order phase-field equation. In the three dimensional case, we prove the existence/uniqueness of local strong solutions for arbitrary initial data and provide some regularity criteria in terms of the velocity field. Then we prove the existence of global strong solutions under the assumption that either the fluid viscosity is large or the initial data is close to an equilibrium. Finally, we investigate the longtime behavior of global solutions and establish stability for local minimizers of the elastic bending energy. Invited Talk: 15:00, August 27 (Wed)

# Nonlinear elliptic problems in a periodic setting

Jaeyoung Byeon

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#### Abstract.

We consider the following equation

$$-\Delta u + A(x)G'(u) = 0 \qquad \text{in} \quad \mathbf{R}^n \tag{1}$$

where A(x) = A(x+i) for  $x \in \mathbf{R}^n$ ,  $i \in \mathbf{Z}^n$ , G(u+j) = G(u) for  $u \in \mathbf{R}$ ,  $j \in \mathbf{Z}$ , G(u) > 0 for  $u \in \mathbf{R} \setminus \mathbf{Z}$ , G(j) = 0, G''(j) > 0 for  $j \in \mathbf{Z}$  and G(t) = G(-t) for  $t \in \mathbf{R}$ . Some recent construction of bounded or unbounded solutions for equation (1) will be introduced.

Invited Talk: 16:00, August 27 (Wed)

# An elementary proof of Strauss conjecture

#### Yi Zhou

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#### Abstract.

We consider the Cauchy problem for the semilinear wave equation and review the solution of the Strauss conjecture which span a time of 30 years. Then we give our recent elementary proof of the global existence part of the Strauss conjecture. Finally, we give open problems for the Schrödinger equations and Klein-Gordon equations.

# Recovery of missing samples in multi-channel oversampling

Kil H. Kwon<sup>2</sup> and Dae Gwan Lee

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**Abstract**. It is well known that in the classical Shannon sampling theory on band-limited signals, any finitely many missing samples can be recovered when the signal is oversampled at a rate higher than the minimum Nyquist rate. In this work, we consider the problem of recovering missing samples from multi-channel oversampling in a general shift-invariant space. We find conditions under which any finite or infinite number of missing samples can be recovered.

 $^{2}$ Speaker

# A Meshless Symplectic Algorithm for multi-variant Hamiltonian PDEs with Radial Basis Approximation

#### Zongmin Wu

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#### Abstract.

Based on radial basis approximation, the paper proposed two methods to discretize the problem of multi-variant Hamiltonian system. One is discretizing the Hamiltonian functional and deriving the corresponding discrete Hamiltonian system. The other is discretizing the system and finding out the corresponding discrete Hamiltonian functional, which will be conserved with respect to the time. This helps open a new area of research in developing the expected meshless symplectic algorithm for multi-variant Hamiltonian systems with the scattered data points. Theoretical estimates including the truncation error and the global error are given. Numerical experiments verify the theoretical results. As numerical experiments show that the schemes are easy to implement with the scattered knots. Furthermore, the successful long-time tracking capability for these Hamiltonian schemes is remarkable.

## A quasi-interpolation scheme for periodic data based on multiquadric trigonometric B-splines

#### Wenwu Gao

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#### Abstract.

Multiquadric (MQ) quasi-interpolation has been studied extensively in the literature. However, the MQ quasi-interpolant is not well defined for periodic data, since its kernel (the MQ function) itself is not periodic. Note that in many applications, the data may arise from a closed curve (surface) and thus possess some kind of periodicity, for example, analysis of geodetic and meteorological data, constructing active contours, estimating the region of attraction of dynamical systems, and so on. Therefore, it is meaningful to construct a quasi-interpolant (whose kernel itself is periodic) for periodic data. In this talk, I shall present the constructions of such a quasi-interpolant. The quasi-interpolant couples together the periodicity of the trigonometric B-spline quasi-interpolant and its derivatives are periodic. The quasi-interpolant covers the trigonometric B-spline quasi-interpolant as a special case. In addition, the error estimate shows that a proper shape parameter can be chosen such that the quasi-interpolant provides the same approximation order as a trigonometric B-spline quasi-interpolant for a periodic between the statemeters and the such that the quasi-interpolant provides the same approximation order as a trigonometric B-spline quasi-interpolant for a periodic function.

# Optimal Throughput and High Short Term Fairness Design of Renewal Access Protocol

Yunbae Kim and Ganguk Hwang<sup>3</sup>

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Abstract. We consider a simple MAC protocol, called the renewal access protocol (RAP), that adopts all of the legacy 802.11 standard but the backoff stage feature. To meet two objectives in the design of the RAP - optimal throughput and high short term fairness, we develop a mathematical model of the RAP and rigorously analyze the performance of the RAP. First, we show that the throughput performance of the RAP depends only on the expectation of the selection distribution where the backoff counter is selected, provided that the number of terminals is fixed, which is in accordance with a well-known result. Second, with the help of renewal and reliability theories we analyze the short term fairness of the RAP. We also show that, if the RAP has a selection distribution of the NBUE (New Better than Used in Expectation) type, the RAP can guarantee high short term fairness. Third, we construct a special binomial distribution which is obviously of the NBUE type that can achieve high short term fairness as well as optimal throughput when used as the selection distribution of the RAP. Furthermore, by the Poisson approximation for binomial distributions, we propose to use in practice a Poisson distribution corresponding to the special binomial distribution. Numerical and simulation results are provided to validate our analysis.

 $^{3}$ Speaker

# Efficient algorithms for computing the non and semi-parametric maximum likelihood estimates with panel count data

#### Liqiang Lu

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#### Abstract.

The maximum likelihood method based on the non-homogeneous Poisson process has been proved an efficient inference procedure for such analysis. However, computing the nonand semiparametric maximum likelihood estimates (MLEs) can be very intensive numerically and the available methods are not efficient. We develop an efficient numerical algorithm stemming from the Newton-Raphson method to compute the non- and semi-parametric MLEs for panel count data. Simulation studies are carried out to demonstrate its numerical efficiency.

# Asymptotics of the solution to the conductivity equation in the presence of adjacent circular inclusions with finite conductivities

#### Mikyoung Lim

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**Abstract**. We consider the conductivity problem in the presence of adjacent circular inclusions with constant conductivities. When two inclusions get closer and their conductivities degenerate to zero or infinity, the gradient of the solution can be arbitrary large. In this talk, I present an asymptotic formula of the solution, which characterizes the gradient blow-up of the solution in terms of conductivities of inclusions as well as the distance between inclusions. The asymptotic formula is expressed in bipolar coordinates in terms of the Lerch transcendent function, and it is valid for inclusions with arbitrary constant conductivities.

Invited Talk: 11:50, August 28 (Thu)

# Cell motility driven by motor proteins

Yunxin Zhang

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#### Abstract.

In this talk, motor protein related cell motility will be discussed. In brief, the following topics will be addressed: (1) mechanism of motor proteins, (2) cargo transportation by multiple motor proteins, (3) motor traffic along microtubule, and (4) dynamic of microtubule driven by motor proteins. This talk will mainly focus on mathematical methods that will be helpful to understand the properties and mechanism cell motility driven by motor proteins.