SHARP TYPE AND COTYPE WITH RESPECT TO QUANTIZED ORTHONORMAL SYSTEMS

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Abstract. Sharp type and cotype of Lebesgue spaces and Schatten classes with respect to quantized orthonormal systems are investigated. This paper complements the question of sharp Fourier type and cotype with respect to compact groups partially solved by recent paper of J. Garcia-Cuerva et al.

1. Introduction

Orthonormal systems can be used to classify Banach spaces according to their geometric properties. Let $(\Omega, \mathcal{M}, \mu)$ be a probability space and $\mathcal{A} = \{a_1, a_2, \cdots\} \subseteq L_2(\Omega)$ be an orthonormal system. For $1 \leq p \leq 2$ and $2 \leq p' \leq \infty$ with $\frac{1}{p} + \frac{1}{p'} = 1$, a Banach space X is said to be $\mathcal{A}$-type p if

$$\Big[\int_\Omega \Big\|\sum_{k=1}^n a_k(\omega) x_k\Big\|_X^{p'} d\mu(\omega)\Big]^{\frac{1}{p'}} \leq C\Big[\sum_{k=1}^n \|x_k\|_X^p\Big]^{\frac{1}{p}}$$

and $\mathcal{A}$-cotype p' if

$$\Big[\int_\Omega \Big\|\sum_{k=1}^n a_k(\omega) x_k\Big\|_X^{p} d\mu(\omega)\Big]^{\frac{1}{p}} \geq C'\Big[\sum_{k=1}^n \|x_k\|_X^{p'}\Big]^{\frac{1}{p'}}$$

for some $C, C' > 0$ and any finite subset $\{x_1, x_2, \cdots\} \subseteq X$.

In other words, X has $\mathcal{A}$-type p if

$$\sup_{n\in\mathbb{N}} \|\mathcal{F}_{\mathcal{A}}^{-1} \otimes I_E\|_{\mathcal{L}(l_p^n(X), L_{p'}(\Omega, X))} < \infty$$

and $\mathcal{A}$-cotype p' if

$$\sup_{n\in\mathbb{N}} \|\mathcal{F}_{\mathcal{A}} \otimes I_E\|_{\mathcal{L}(L^n_{p'}(\Omega, X), l_p^n(X))} < \infty,$$

where $L^n_p(\Omega, X) = \mathrm{span}\{a_1, a_2, \cdots, a_n\} \otimes X \subseteq L_p(\Omega, X)$ and

$$\mathcal{F}_{\mathcal{A}}(f)(n) = \int_\Omega f(\omega)\overline{a_n(\omega)} d\mu(\omega) \ \text{ and } \ \mathcal{F}_{\mathcal{A}}^{-1}((x_k))(\omega) = \sum_k a_k(\omega) x_k$$

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for appropriate $f : \Omega \to \mathbb{C}$ and finite sequence (x_k) in X. When we consider the classical Rademacher system $\mathcal{R} = \{r_1, r_2, \cdots\}$, where $r_k(t) = \mathrm{sign}(\sin(2^k \pi t))$, $t \in [0,1]$ and $k = 1, 2, \cdots$, we get the usual type and cotype, and the trigonometric system $\mathcal{E} = \{e^{2\pi i n} : n \in \mathbb{Z}\}$ gives the Fourier type([5]).

As a noncommutative analogue of the above, the notion of type and cotype of operator spaces with respect to quantized orthonormal systems was introduced and investigated in the recent paper [2] of J. Garcia-Cuerva et al.

Definition 1. *For a probability space $(\Omega, \mathcal{M}, \mu)$ with no atoms and a family of positive integers $\mathbf{d}_\Sigma = \{d_\pi : \pi \in \Sigma\}$ indexed by Σ, a collection of matrix-valued functions $\mathcal{A} = \{\varphi^\pi : \Omega \to M_{d_\pi}\}_{\pi \in \Sigma}$ with measurable entries is said to be a uniformly bounded quantized orthonormal system(u.b.q.o.s. for short) if the following conditions holds:*

$$\int_\Omega \varphi^\pi_{ij} \varphi^{\pi'}_{i'j'}(\omega) d\mu(\omega) = \frac{1}{d_\pi} \delta_{\pi,\pi'} \delta_{i,i'} \delta_{j,j'}; \tag{1}$$

$$\sup_{\pi \in \Sigma} \|\varphi^\pi\|_{L_\infty(\mu, S^{d_\pi}_\infty)} = M_{\mathcal{A}} < \infty. \tag{2}$$

A typical example of u.b.q.o.s. is the dual object $\widehat{G}$ of a compact group G with $\Sigma = \widehat{G}$, d_π is the dimension of a irreducible representation $\pi \in \widehat{G}$ of G and $\varphi^\pi = \pi$. Another important example is the quantized Rademacher system $\mathcal{R}_\Sigma$ associated to the index set Σ in Definition 1, which is the collection of independent random variables $\epsilon^\pi : G \to \mathcal{O}(d_\pi)$ where $\pi \in \Sigma$ and the distribution of ϵ^π is exactly the normalized Haar measure on the orthogonal group $\mathcal{O}(d_\pi)$.

Now for a u.b.q.o.s. $\mathcal{A}$ we consider the following transforms as in the commutative case:

$$\mathcal{F}_{\mathcal{A}}(f)(\pi) = \int_\Omega f(\omega) \varphi^\pi(\omega)^* d\mu(\omega) \quad \text{and} \quad \mathcal{F}^{-1}_{\mathcal{A}}(A)(\omega) = \sum_{\pi \in \Sigma} d_\pi \mathrm{tr}(A^\pi \varphi^\pi(\omega))$$

for appropriate $f : \Omega \to \mathbb{C}$ and $A \in \prod_{\pi \in \Sigma} M_{d_\pi}$.

Definition 2. *For $1 \le p \le 2$ and $2 \le p' \le \infty$ with $\frac{1}{p} + \frac{1}{p'} = 1$, we say that an operator space E has $\mathcal{A}$-type p if*

$$\|E|\mathcal{A}\mathcal{T}_p\| = \sup_{\text{finite}\, \Gamma \subseteq \Sigma} \left\|\mathcal{F}^{-1}_{\mathcal{A}} \otimes I_E\right\|_{cb(\mathcal{L}_p(\Gamma, E), L_{p'}(\Omega, E))} < \infty$$

and that E has $\mathcal{A}$-cotype p' if

$$\|E|\mathcal{A}\mathcal{C}_{p'}\| = \sup_{\text{finite}\, \Gamma \subseteq \Sigma} \|\mathcal{F}_{\mathcal{A}} \otimes I_E\|_{cb(L^\Gamma_p(\Omega, E), \mathcal{L}_{p'}(\Gamma, E))} < \infty,$$

where $L_p^\Gamma(\Omega, E) = \text{span}\{\varphi_{ij}^\pi : \pi \in \Gamma\} \otimes E \subseteq L_p(\Omega, E)$ *and* $\mathcal{L}_p(\Gamma, E)$ *is the vector-valued noncommutative* L_p*-space defined as follows:*

$$\mathcal{L}_q(\Gamma, E) = \{A \in \prod_{\pi\in\Gamma} M_{d_\pi} \otimes E : \|A\|_{\mathcal{L}_q(\Gamma,E)} = \Big(\sum_{\pi\in\Gamma} d_\pi \|A^\pi\|^q_{S_q^{d_\pi}(E)}\Big)^{\frac{1}{q}} < \infty\}$$

for $1 \leq q < \infty$ *and*

$$\mathcal{L}_\infty(\Gamma, E) = \{A \in \prod_{\pi\in\Gamma} M_{d_\pi} \otimes E : \|A\|_{\mathcal{L}_\infty(\Gamma,E)} = \sup_{\pi\in\Gamma} \|A^\pi\|_{S_\infty^{d_\pi}(E)} < \infty\}.$$

See chapter 2 of [6] for the details of $\mathcal{L}_p(\Gamma, E)$.

It is well known that l_p and Schatten von-Neumann class S_p $(1 \leq p < \infty)$ has (Rademacher) type $\min(p, 2)$ and cotype $\max(2, p')$ and can not have better type and cotype as Banach spaces, that is, $\min(p, 2)$ and $\max(2, p')$ are '*sharp*' type and cotype of l_p and S_p respectively. When $p = \infty$, l_∞ and S_∞ does not have nontrivial type and cotype. It is also well known that l_p and S_p $(1 \leq p \leq\leq \infty)$ have '*sharp*' Fourier type $\min(p, p')$.([5]) Thus it is natural to be interested in whether we can determine sharp $\mathcal{A}$-type and $\mathcal{A}$-cotype for l_p and S_p as operator spaces.

In [1, 2], it is shown that l_p and $S_p(1 \leq p \leq \infty)$ has $\mathcal{A}$-type $\min(p, p')$ and $\mathcal{A}$-cotype $\max(p, p')$ for any u.b.q.o.s. $\mathcal{A}$. The sharpness of this type and cotype is partially answered in [3]. For an infinite compact semisimple Lie group G, it is shown that l_p and S_p can not have $\widehat{G}$-cotype q' and $l_{p'}$ and $S_{p'}$ can not have $\widehat{G}$-type q for $1 \leq p < q \leq 2$ in [3]. Note that $\widehat{G}$-type q and $\widehat{G}$-cotype q' are exactly the same with Fourier cotype q' and Fourier type q with respect to G in [3] respectively.

In this paper, we consider the dual case of the above in more general situation. We prove that l_p can not have $\mathcal{A}$-type q and $l_{p'}$ can not have $\mathcal{A}$-cotype q' for any infinite u.b.q.o.s. $\mathcal{A}$ (which means that Σ is infinite) and $1 \leq p < q \leq 2$ by checking the same statement for the corresponding quantized Rademacher system $\mathcal{R}_\Sigma$ and reminding the fact that $\mathcal{A}$-type q and $\mathcal{A}$-cotype q' imply $\mathcal{R}_\Sigma$-type q and $\mathcal{R}_\Sigma$-cotype q' respectively (Proposition 3.5 in [2]). Using this result, we prove that $\min(p, p')$ is the sharp $\mathcal{A}$-type of S_p, and $\max(p, p')$ is the sharp $\mathcal{A}$-cotype for any infinite u.b.q.o.s. $\mathcal{A}$ and $1 \leq p \leq \infty$, which improves the result in [3] for S_p. Note that since every infinite dimensional L_p-space on a σ-finite measure space has the same local structure with l_p, we have the same sharp type and cotype results for such L_p-spaces.

2. Sharp type and cotype of l_p

From now on let $\mathcal{A}$ be an infinite u.b.q.o.s. as in Definition 1, $\mathcal{R} = \mathcal{R}_\Sigma$ be the corresponding quantized Rademacher system and p, p', q and q' be the fixed exponents satisfying $1 \leq p < q \leq 2$, $\frac{1}{p} + \frac{1}{p'} = 1$, and $\frac{1}{q} + \frac{1}{q'} = 1$.

Theorem 3. *l_p can not have $\mathcal{A}$-type q and $l_{p'}$ can not have $\mathcal{A}$-cotype q'.*

Proof. Suppose that l_p has $\mathcal{A}$-type q, then l_p has $\mathcal{R}$-type q by Proposition 3.5 in [2], which is equivalent to

$$\mathcal{F}_{\mathcal{R}}^{-1} \otimes I_{l_p} : \mathcal{L}_q(\Gamma, l_p) \to L_{q'}(\Omega, l_p)$$

is uniformly completely bounded for any finite $\Gamma \subseteq \Sigma$, then we have that

$$\mathcal{F}_{\mathcal{R}}^{-1} \otimes I_{l_p} : \mathcal{L}_q(\Gamma, l_p) \to L_p(\Omega, l_p)$$

is bounded with norm smaller than or equal to $\|l_p|\mathcal{RT}_q\|$ since Ω is a probability space.

Let's fix $\Gamma = \{\pi_1, \pi_2, \cdots, \pi_n\}$, and let $N = \sum_{k=1}^n d_{\pi_k} = \sum_{k=1}^n d_k$. Then we have for all $A \in \mathcal{L}_q(\Gamma, l_p)$,

$$\begin{aligned} \left\|\mathcal{F}_{\mathcal{R}}^{-1} \otimes I_{l_p}(A)\right\|_{L_p(\Omega, l_p)} &\leq \|l_p|\mathcal{RT}_q\| \cdot \|A\|_{\mathcal{L}_q(\Gamma, l_p)} \\ &\leq N^{\frac{1}{q}-\frac{1}{2}} \cdot \|l_p|\mathcal{RT}_q\| \cdot \|A\|_{\mathcal{L}_2(\Gamma, l_p)}. \end{aligned} \tag{3}$$

Consider $A \in \mathcal{L}_2(\Gamma, l_p)$ be given by $A^k (= A^{\pi_k}) = \mathrm{diag}(\alpha_k e_{k,i}) \in M_{d_k}(l_p^n)$, $1 \leq k \leq n$, where $\alpha_k \geq 0$ will be fixed later, and $e_{k,i}$'s are distinct unit vectors in l_p^N. Then we get

$$\begin{aligned} \mathcal{F}_{\mathcal{R}}^{-1} \otimes I_{l_p}(A)(\omega) &= \sum_{k=1}^n d_k \mathrm{tr}(A^k \epsilon^k(\omega)) \\ &= \sum_{k=1}^n \sum_{i=1}^{d_k} d_k \alpha_k \epsilon_{ii}^k(\omega) e_{k,i} \end{aligned}$$

for $\omega \in \Omega$, and by applying the noncommutative version of Khintchine's inequality(Corollary 2.12 of [4]), we get for some constant $C_1 > 0$,

$$\begin{aligned} \left\|\mathcal{F}_{\mathcal{R}}^{-1} \otimes I_{l_p}(A)\right\|_{L_p(\Omega, l_p)} &= \Big(\sum_{k=1}^n \sum_{i=1}^{d_k} d_k^p \alpha_k^p \Big[\int_\Omega \left|\epsilon_{ii}^k(\omega)\right|^p d\mu(\omega)\Big]\Big)^{\frac{1}{p}} \\ &\geq C_1 \Big(\sum_{k=1}^n \sum_{i=1}^{d_k} d_k^p \alpha_k^p \Big[\int_\Omega \left|\epsilon_{ii}^k(\omega)\right|^2 d\mu(\omega)\Big]^{\frac{p}{2}}\Big)^{\frac{1}{p}} \\ &= C_1 \Big(\sum_{k=1}^n \sum_{i=1}^{d_k} d_k^p \alpha_k^p d_k^{-\frac{p}{2}}\Big)^{\frac{1}{p}} \\ &= C_1 \Big(\sum_{k=1}^n d_k^{1+\frac{p}{2}} \alpha_k^p\Big)^{\frac{1}{p}}. \end{aligned} \tag{4}$$

Since A^k's are diagonal matrices, by Corollary 1.3 in [6], we have that

$$\|A\|_{\mathcal{L}_2(\Gamma, l_p)} = \Big(\sum_{k=1}^n d_k \sum_{i=1}^{d_k} \alpha_k^2\Big)^{\frac{1}{2}} = \Big(\sum_{k=1}^n d_k^2 \alpha_k^2\Big)^{\frac{1}{2}}. \tag{5}$$

If we choose α_k's such that $d_k^{\frac{p}{2}}\alpha_k^p = D$ for a constant $D > 0$, then we get

$$\frac{\left\|\mathcal{F}_{\mathcal{R}}^{-1} \otimes I_{l_p}(A)\right\|_{L_p(\Omega,l_p)}}{\|A\|_{\mathcal{L}_2(\Gamma,l_p)}} \geq C_1\Big(\sum_{k=1}^n d_k\Big)^{\frac{1}{p}-\frac{1}{2}} = C_1 N^{\frac{1}{p}-\frac{1}{2}}. \tag{6}$$

Combining (3) and (6), we get

$$\|l_p|\mathcal{RT}_q\| \geq C_1 N^{\frac{1}{p}-\frac{1}{q}},$$

which is contradictory since we can choose finite $\Gamma \subseteq \Sigma$ so that N is arbitrarily large.

We proceed similarly for the $l_{p'}$ case. Suppose that $l_{p'}$ has $\mathcal{A}$-cotype q', then $l_{p'}$ has $\mathcal{R}$-cotype q', and we have that for any finite $\Gamma \subseteq \Sigma$,

$$\mathcal{F}_{\mathcal{R}} \otimes I_{l_{p'}} : L^{\Gamma}_{p'}(\Omega, l_{p'}) \to \mathcal{L}_{q'}(\Gamma, l_{p'})$$

is completely bounded with c.b. norm smaller than or equal to $\|l_{p'}|\mathcal{RC}_{q'}\|$ since Ω is a probability space. Thus we have that for all $A \in \mathcal{L}_{q'}(\Gamma, l_{p'})$,

$$\begin{aligned} \|l_{p'}|\mathcal{RC}_{q'}\| \cdot \left\|\mathcal{F}_{\mathcal{R}}^{-1} \otimes I_{l_{p'}}(A)\right\|_{L^{\Gamma}_{p'}(\Omega,l_{p'})} &\geq \|A\|_{\mathcal{L}_{q'}(\Gamma,l_{p'})} \\ &\geq N^{\frac{1}{q'}-\frac{1}{2}} \cdot \|A\|_{\mathcal{L}_2(\Gamma,l_{p'})}. \end{aligned} \tag{7}$$

Now we consider the same $A \in \mathcal{L}_2(\Gamma, l_{p'})$ given by $A^k (= A^{\pi_k}) = \mathrm{diag}(\alpha_k e_{k,i}) \in M_{d_k}(l_p^n)$, $1 \leq k \leq n$ as in the above.

When $p' < \infty$, by the same calculation we get for some constant $C_2 > 0$,

$$\frac{\|A\|_{\mathcal{L}_2(\Gamma,l_{p'})}}{\left\|\mathcal{F}_{\mathcal{R}}^{-1} \otimes I_{l_{p'}}(A)\right\|_{L^{\Gamma}_{p'}(\Omega,l_{p'})}} \geq C_2\Big(\sum_{k=1}^n d_k\Big)^{\frac{1}{2}-\frac{1}{p'}} = C_2 N^{\frac{1}{2}-\frac{1}{p'}}. \tag{8}$$

Combining (7) and (8), we get

$$\|l_{p'}|\mathcal{RC}_{q'}\| \geq C_2 N^{\frac{1}{q'}-\frac{1}{p'}} = C_2 N^{\frac{1}{p}-\frac{1}{q}},$$

which is also contradictory since we can choose finite $\Gamma \subseteq \Sigma$ so that N is arbitrarily large.

When $p' = \infty$, we recall that l_∞ contains isomorphic copies of l_r for all $1 \leq r < \infty$ as subspaces. Since we used norms of l_p only in (4) and (5), and furthermore, the calculation depends only on the norm of l_p as a Banach space, l_∞ cannot have finite cotype.

□

We say that an operator E has Banach-$\mathcal{A}$-type p if

$$\sup_{\text{finite}\,\Gamma\subseteq\Sigma} \left\|\mathcal{F}_{\mathcal{A}}^{-1} \otimes I_E\right\|_{\mathcal{L}_p(\Gamma,E)\to L_{p'}(\Omega,E)} < \infty.$$

Banach-$\mathcal{A}$-cotype p' is defined similarly. If we look at the proof of Proposition 3.5 in [2], we can easily see that Banach-$\mathcal{A}$-type p and Banach-$\mathcal{A}$-cotype p' imply Banach-$\mathcal{R}$-type p and Banach-$\mathcal{R}$-cotype p' respectively. Thus by the same observation we used for l_∞ at the end of the proof of Theorem 3, we can present a modification of Theorem 3, which will be used in the next section.

Theorem 4. *Let E be an operator space such that the underlying Banach space contains an isomorphic copy of l_p. Then*

$$\mathcal{F}_{\mathcal{A}}^{-1} \otimes I_E : \mathcal{L}_q(\Gamma, E) \to L_{q'}(\Omega, E)$$

is not uniformly bounded for all finite $\Gamma \subseteq \Sigma$.
Similarly, if the underlying Banach space of E contains an isomorphic copy of $l_{p'}$, then

$$\mathcal{F}_{\mathcal{A}} \otimes I_E : L_q^\Gamma(\Omega, E) \to \mathcal{L}_{q'}(\Gamma, E)$$

is not uniformly bounded for all finite $\Gamma \subseteq \Sigma$.

3. Sharp type and cotype of S_p

Since $S_r (1 \leq r \leq \infty)$ contains l_r as a subspace, we have the following half results easily from Theorem 3.

Theorem 5. *S_p can not have $\mathcal{A}$-type q and $S_{p'}$ can not have $\mathcal{A}$-cotype q'.*

Theorem 4 enables us to answer the remaining half of sharp type and cotype of S_p.

Theorem 6. *S_p can not have $\mathcal{A}$-cotype q' and $S_{p'}$ can not have $\mathcal{A}$-type q.*

Proof. Suppose that S_p have $\mathcal{A}$-cotype q', which is equivalent to

$$\mathcal{F}_{\mathcal{A}} \otimes I_{S_p} : L_q^\Gamma(\Omega, S_p) \to \mathcal{L}_{q'}(\Gamma, S_p)$$

is uniformly completely bounded for all finite $\Gamma \subseteq \Sigma$. Then we have that

$$I_{S_{q'}} \otimes \mathcal{F}_{\mathcal{A}} \otimes I_{S_p} : S_{q'}(L_q^\Gamma(\Omega, S_p)) \to S_{q'}(\mathcal{L}_{q'}(\Gamma, S_p))$$

is uniformly bounded by Lemma 1.7 of [6] and consequently

$$\mathcal{F}_{\mathcal{A}} \otimes I_{S_{q'}(S_p)} : L_q^\Gamma(\Omega, S_{q'}(S_p)) \to \mathcal{L}_{q'}(\Gamma, S_{q'}(S_p))$$

is uniformly bounded by Corollary 1.10 and Proposition 2.1 of [6].

However, Theorem 1.1 of [6] implies that

$$\begin{aligned} S_{q'}(S_p) &= R(1/q) \otimes_h S_p \otimes_h R(1/q') \\ &= R(1/q) \otimes_h R(1/p') \otimes_h R(1/p) \otimes_h R(1/q'), \end{aligned}$$

which means that $R(1/q) \otimes_h R(1/p')$ is a subspace of $S_{q'}(S_p)$, where

$$\begin{aligned} R(1/q) \otimes_h R(1/p') &= [R \otimes_h R(1/p'), C \otimes_h R(1/p')]_{\frac{1}{q}} \\ &= [[R \otimes_h R, R \otimes_h C]_{\frac{1}{p'}}, [C \otimes_h R, C \otimes_h C]_{\frac{1}{p'}}]_{\frac{1}{q}}. \end{aligned}$$

Since $R \otimes_h R$ and $C \otimes_h C$ are isometric to S_2 and $R \otimes_h C$ and $C \otimes_h R$ are completely isometric to S_1 and S_∞ respectively, we have that $R(1/q) \otimes_h R(1/p')$ is isometric to S_r with $\frac{1}{r} = \frac{1}{2}(\frac{1}{p'} + \frac{1}{q'}) < \frac{1}{q'}$. This implies

$$\mathcal{F}_{\mathcal{A}} \otimes I_{S_{q'}(S_p)} : L_q^\Gamma(\Omega, S_{q'}(S_p)) \to \mathcal{L}_{q'}(\Gamma, S_{q'}(S_p))$$

is not uniformly bounded for all finite $\Gamma \subseteq \Sigma$ by Theorem 4, which lead us to a contradiction. The type case is obtained similarly.

□

Remark 7. By the same observation in the proof of Theorem 3, l_∞ does not have nontrivial $\mathcal{A}$-type also. However, we don't know whether l_p has $\mathcal{A}$-cotype q' and $l_{p'}(p' < \infty)$ has $\mathcal{A}$-type q even in the case $\mathcal{A} = \mathcal{R}$. The best result we can show is that

$$\mathcal{F}^{-1}_{\mathcal{R}_G} \otimes I_{l_{p'}} : \mathcal{L}_2(\Gamma, l_{p'}) \to L_2(\Omega, l_{p'})$$

and

$$\mathcal{F}_{\mathcal{R}_G} \otimes I_{l_p} : L_2^\Gamma(\Omega, l_p) \to \mathcal{L}_2(\Gamma, l_p)$$

are uniformly bounded for all finite $\Gamma \subseteq \Sigma$.

We check the first statement only because l_p case is obtained similarly. Now we consider $A \in \mathcal{L}_2(\Gamma, l_{p'})$. Then we have

$$\begin{aligned}
\left\| \mathcal{F}^{-1}_{\mathcal{R}_G} \otimes I_{l_{p'}}(A) \right\|_{L_2(\Omega, l_{p'})} &\leq \left\| \mathcal{F}^{-1}_{\mathcal{R}_G} \otimes I_{l_{p'}}(A) \right\|_{L_{p'}(\Omega, l_{p'})} \\
&= \Big[\int_\Omega \Big\| \sum_{\pi \in \Gamma} d_\pi \mathrm{tr}(A^\pi \epsilon^\pi(\omega)) \Big\|^{p'}_{l_{p'}} d\mu(\omega) \Big]^{\frac{1}{p'}} \\
&= \Big[\sum_{i=1}^\infty \int_\Omega \Big| \sum_{\pi \in \Gamma} d_\pi \mathrm{tr}(A_i^\pi \epsilon^\pi(\omega)) \Big|^{p'} d\mu(\omega) \Big]^{\frac{1}{p'}},
\end{aligned}$$

where A_i^π implies i-th component of A^π as an element in $l_{p'}(M_{d_\pi}) (= M_{d_\pi}(l_{p'}))$. By applying the noncommutative version of Khintchine's inequality, we get for some constant $C > 0$,

$$\begin{aligned}
\left\| \mathcal{F}^{-1}_{\mathcal{R}_G} \otimes I_{l_{p'}}(A) \right\|_{L_2(\Omega, l_{p'})} &\leq C \Big[\sum_{i=1}^\infty \Big(\int_\Omega \Big| \sum_{\pi \in \Gamma} d_\pi \mathrm{tr}(A_i^\pi \epsilon^\pi(\omega)) \Big|^2 d\mu(\omega) \Big)^{\frac{p'}{2}} \Big]^{\frac{1}{p'}} \\
&= C \Big[\sum_{i=1}^\infty \Big(\sum_{\pi \in \Gamma} d_\pi \left\| A_i^\pi \right\|^2_{S_2} \Big)^{\frac{p'}{2}} \Big]^{\frac{1}{p'}} \\
&= C \left\| (A_i^\pi)_{i=1}^\infty \right\|_{l_{p'}(\mathcal{L}_2(\Gamma))} \leq C \left\| (A_i^\pi)_{i=1}^\infty \right\|_{\mathcal{L}_2(\Gamma, l_{p'})}.
\end{aligned}$$

The last line is by Corollary 1.10 of [6] again.

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