OPERATORS OF FOURIER TYPE p WITH RESPECT TO SOME SUBGROUPS OF A LOCALLY COMPACT ABELIAN GROUP*

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Abstract. It is shown that all Banach space operators which have Fourier type $p\,(1 < p \leq 2)$ with respect to a second countable locally compact abelian group G also have Fourier type p with respect to every closed discrete subgroup H of G. And the same statement holds for any closed subgroup H of G when $p = 2$. Also shown as a corollary is that Fourier type 2 operators have Walsh type 2.

1. Introduction

In geometry of Banach spaces, it is of great importance to be able to extend scalar-valued analysis to vector-valued setting. We can classify Banach spaces and their operators by measuring how well those scalar-valued analysis problems can be extended. For example, J. Peetre[9] considered the concept of Fourier type p, for $1 < p \leq 2$, of a Banach space; he proved that the vector-valued Fourier transform on $\mathbb{R}$ is a well-defined bounded linear operator from $L_p(\mathbb{R}, X)$ to $L_{p'}(\mathbb{R}, X)$. Here p' is the Hölder conjugate of p, i.e. $2 \leq p < \infty$ with $\frac{1}{p} + \frac{1}{p'} = 1$. And M. Milman[8] later extended that concept to Fourier type p, for $1 < p \leq 2$, with respect to a locally compact abelian group G; namely, the vector-valued Fourier transform on G is a well-defined bounded linear operator from $L_p(G, X)$ to $L_{p'}(G', X)$ where G' is the dual group of G. We are going to concentrate on the latter subject in this paper.

Since the work of M. Milman[8], many researchers have deepened the theory of the relationship between the structures of groups and the geometry of Banach spaces. H. König[6] showed that $\mathbb{T}$, $\mathbb{R}$ and $\mathbb{Z}$ give the same classification; in other words, an operator T has Fourier type p with respect to $\mathbb{T}$ iff T has Fourier type p with respect to $\mathbb{R}$ iff T has Fourier type p with respect to $\mathbb{Z}$. And M. E. Andersson[1] showed that $\mathbb{Z}$ and $\mathbb{Z}^n$ are equivalent in the above sense for all $n \in \mathbb{N} \cup \{\infty\}$ and that if H is an open subgroup of a locally compact abelian

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group G, then all Banach space operators which have Fourier type $p\,(1<p\leq 2)$ with respect to G also have Fourier type p with respect to H (Actually he proved this fact only for identity operators T but one can easily see that the general result holds for operators also by observing the proof carefully).

We prove here that all Banach space operators which have Fourier type $p\,(1<p\leq 2)$ with respect to a second countable locally compact abelian group G also have Fourier type p with respect to every closed discrete subgroup H of G, and the same statement holds for any closed subgroup H of G when $p=2$. We also obtain, as a corollary, the fact that Fourier type 2 operator has Walsh type 2, which has been already proved by A. Hinrichs[5].

In section 2, we will present various well known facts needed in the proof of our main results. In section 3.1, we prove one of our main results. Finally, in section 3.2, the case $p=2$ of our result and its applications are presented.

2. Preliminaries

We need the following well known results about locally compact abelian groups. We write LCA for 'locally compact abelian' and we denote by G' the dual group of a LCA group G from now on. Most facts in this section are found in monographs [12], [2] and [4].

Proposition 1. (Weil's formula) *Let G be a LCA group and H a closed subgroup of G. For any Haar measures μ_G and μ_H on G and H, respectively, there exists a unique Haar measure $\mu_{G/H}$ on G/H such that for every $f\in C_c(G)$*

$$\int_G f(x)d\mu_G(x)=\int_{G/H}\int_H f(xh)d\mu_H(h)d\mu_{G/H}(xH).$$

Proof. See p.57 of [2]. □

Proposition 2. *The same formula in Proposition 1 holds for every lower semi-continuous $f:G\to[0,\infty]$ and for every $f\in L_1(G)$.*

Proof. See p.62 of [2] and [11]. □

Although integral form of Weil's formula looks similar to Fubini's theorem, it is not exactly Fubini's theorem. Therefore, we need some additional condition in order to reduce it to Fubini's theorem.

Proposition 3. (The Borel selection lemma) *Let G be a second countable LCA group and H a closed subgroup of G. Then there exist a Borel set $A\subseteq G$ such* *that A meets each coset of H at exactly one point and the following two functions are measurable bijections:*

$$\begin{aligned} q|_A : A \subseteq G \to G/H, \quad & \phi : A \times H \to G. \\ a \mapsto aH \qquad\qquad & (a,h) \mapsto ah \end{aligned}$$

We call A a Borel selection for q where q is the canonical quotient map from G onto G/H.

Proof. See lemma 1.1 and 1.2 of [7]. □

Proposition 4. (Fourier inversion formula) *Let G be a LCA group. Then there exists a unique Haar measure $\mu_{G'}$ on G' such that the following inversion formula holds*

$$f(x) = \int_{G'} \gamma(x)\widehat{f}(\gamma)d\mu_{G'}(\gamma)$$

almost all $x \in G$, for all $f \in L_1(G)$ which satisfies $\widehat{f} \in L_1(G')$. This unique Haar measure $\mu_{G'}$ is called the dual measure of μ_G.

Proof. See p.97 of [2]. □

Proposition 5. (Plancherel's theorem) *Let G be a LCA group and μ_G be a Haar measure on G. Let $\mu_{G'}$ be the dual measure of μ_G. Then we have the following isometry:*

$$\|f\|_{L_2(G,\mu_G)} = \left\|\widehat{f}^G\right\|_{L_2(G',\mu_{G'})}$$

for all $f \in L_2(G, \mu_G)$.

Proof. See p.99 of [2]. □

Now consider a LCA group G and a closed subgroup H of G. And let μ_G and μ_H be given Haar measures on G and H, respectively. Then by Proposition 4, we can think of $\mu_{G'}$ and $\mu_{H'}$. Also at the same time, we also can consider $\mu_{G/H}$ by Proposition 1. Since $H^\perp \cong (G/H)'$ we set $\mu_{H^\perp}$ as $\mu_{(G/H)'}$ obtained by Proposition 4. Then we get another natural measure $\mu_{G'/H^\perp}$ on $G'/H^\perp \cong H'$. Therefore, we have to deal with the compatibility problem of $\mu_{H'}$ and $\mu_{G'/H^\perp}$.

Proposition 6. *The measures $\mu_{H'}$ and $\mu_{G'/H^\perp}$ mentioned above are actually the same.*

Proof. See 5.5.4 of [11]. □

Finally we close this section by reiterating the definition of Fourier type of a Banach space operator with respect to a LCA group.

Definition 1. Let G be a LCA group and $T : X \to Y$ be a bounded linear Banach space operator. Let μ_G be a Haar measure on G and $\mu_{G'}$ be the dual measure of μ_G. Then T is said to have Fourier $p\,(1 < p \leq 2)$ with respect to G and we write $T \in \mathcal{FT}_p^G$ if the operator $[F^G, T]$ given by

$$f(x) \to T\widehat{f}^G(\gamma) := \int_G Tf(x)\overline{\gamma(x)}d\mu_G(x)$$

defines a bounded linear operator from $L_p(G, X)(= L_p(G, \mu_G, X))$ to $L_{p'}(G', Y)(= L_{p'}(G', \mu_{G'}, Y))$. In this case, we set

$$\left\|T|\mathcal{FT}_p^G\right\| = \left\|[F^G, T] : L_p(G, X) \to L_{p'}(G', Y)\right\|.$$

Remark 1. In the above definition, the norm $\left\|T|\mathcal{FT}_p^G\right\|$ is independent of the choice of μ_G since the dual measure of $\lambda\mu_G$ is $\lambda^{-1}\mu_{G'}$ for any $\lambda > 0$.

3. Some subgroups which inherit the fourier type properties from their supergroups

3.1. The case $1 < p \leq 2$.

Theorem 7. *Let G be a second countable LCA group, H be a closed discrete subgroup of G and $1 < p \leq 2$. Then there is a constant $C_p > 0$ such that for any Banach space operator $T : X \to Y$ we have*

$$\left\|T|\mathcal{FT}_p^H\right\| \leq C_p \left\|T|\mathcal{FT}_p^G\right\|.$$

Proof. Let μ_G and μ_H be given Haar measures on G and H, respectively. Then we can consider $\mu_{G/H}$ by Proposition 1. And let $A \subseteq G$, $q|_A$ and ϕ be the same as in Proposition 3. Now let μ_A be the measure on A induced by $q|_A$ and choose a compact subset $E \subseteq G/H$ such that $\mu_{G/H}(E) = \epsilon > 0$. Then $B = {q|_A}^{-1}(E)$ is a Borel subset of A with $\mu_A(B) = \epsilon$. To finish the proof, we need to find a constant $C_p > 0$ such that

$$\left\|T\widehat{f}^H\right\|_{L_{p'}(H', Y)} \leq C_p \left\|T|\mathcal{FT}_p^G\right\| \cdot \|f\|_{L_p(H,X)} \tag{1}$$

for all $f \in L_p(H, X)$.

Let us consider functions f of the following form:

$$f(h) = \sum_{n=1}^{N} v_n f_n(h)$$

where N is a positive integer, $v_n \in X$ and $f_n \in C_c(H)$. It suffices to check (1) for these functions since such functions are dense in $L_p(H, X)$.

Now consider an extension g of f to whole G by defining for all $x = ah \in G$

$$g(x) = f(h)1_B(a)$$

where $a \in A$, $h \in H$ and 1_B is the characteristic function on B.

Then, since for all $\lambda > 0$

$$\begin{aligned}\phi^{-1}\{x : g(x) > \lambda\} &= \{(a,h) : f(h)1_B(a) > \lambda\} \\ &= B \times \{h : f(h) > \lambda\} \subseteq A \times H,\end{aligned}$$

we easily see that g is Borel measurable in G and furthermore, by applying Weil's formula to $1_{\{x:g(x)>\lambda\}}$, we get

$$\begin{aligned}\int_G 1_{\{g>\lambda\}}(x)d\mu_G(x) &= \int_G 1_{\{g>\lambda\}}(ah)d\mu_G(ah) \\ &= \int_{G/H}\int_H 1_B(a)1_{\{f>\lambda\}}(\xi h)d\mu_H(\xi)d\mu_{G/H}(ahH) \\ &= \int_{G/H}\int_H 1_{q(B)}(aH)1_{\{f>\lambda\}}(\xi)d\mu_H(\xi)d\mu_{G/H}(aH) \\ &= \epsilon\int_H 1_{\{f>\lambda\}}(\xi)d\mu_H(\xi).\end{aligned}$$

Hence it follows that

$$\|g\|_{L_r(G,X)} = \epsilon^{\frac{1}{r}}\,\|f\|_{L_r(H,X)}$$

for all $1 \leq r < \infty$.

Observe that $g = \sum_{n=1}^{N} v_n g_n$ is a finite linear combination of $L_1(G)$ functions with vector coefficients, where g_n's are the extensions of f_n just like that g is the extension of f. Here $g \in L_1(G)$, hence we can apply Weil's formula to g.

Thus, we have for $\gamma \in G'$,

$$\begin{aligned}\widehat{g}^G(\gamma) &= \int_G g(x)\overline{\gamma(x)}d\mu_G(x) \\ &= \int_G 1_B(a)f(h)\overline{\gamma(ah)}d\mu_G(ah) \\ &= \int_{G/H} 1_{q(B)}(aH)\int_H f(h\xi)\overline{\gamma(ah\xi)}d\mu_H(\xi)d\mu_{G/H}(ahH) \\ &= \int_A 1_B(a)\int_H f(h\xi)\overline{\gamma(ah\xi)}d\mu_H(\xi)d\mu_A(a) \\ &= \Big[\int_A 1_B(a)\overline{\gamma(a)}d\mu_A(a)\Big]\cdot\Big[\int_H f(\xi)\overline{\gamma|_H(\xi)}d\mu_H(\xi)\Big].\end{aligned}$$

Since $\left\|\widehat{g}^G\right\|^{p'}$ is continuous on G' and nonnegative, we get the followings by applying Weil's fomula again:

$$\begin{aligned}
&\left\|T\widehat{g}^G\right\|^{p'}_{L_{p'}(G',Y)}\\
&=\int_{G'}\left\|T\widehat{g}^G(\gamma)\right\|^{p'}d\mu_{G'}(\gamma)\\
&=\int_{G'}\Big|\int_B\overline{\gamma(a)}d\mu_A(a)\Big|^{p'}\cdot\left\|T\widehat{f}^H(\gamma|_H)\right\|^{p'}d\mu_{G'}(\gamma)\\
&=\int_{H'}\int_{H^\perp}\Big|\int_B\overline{\gamma(a)\psi(a)}d\mu_A(a)\Big|^{p'}\cdot\left\|T\widehat{f}^H(\gamma\cdot\psi|_H)\right\|^{p'}d\mu_{H^\perp}(\psi)d\mu_{H'}(\gamma H^\perp)\\
&=\int_{H'}\Big[\int_{H^\perp}\Big|\int_B\overline{\gamma(a)\psi(a)}d\mu_A(a)\Big|^{p'}d\mu_{H^\perp}(\psi)\Big]\cdot\left\|T\widehat{f}^H(\gamma|_H)\right\|^{p'}d\mu_{H'}(\gamma H^\perp).
\end{aligned}$$

Now we claim that the infimum of

$$\int_{H^\perp}|\int_B\overline{\gamma(a)\psi(a)}d\mu_A(a)|^{p'}d\mu_{H^\perp}(\psi) \tag{2}$$

over $\gamma|_H\in H'$ is positive.

First, let's show that

$$\begin{aligned}
\Phi:&H'\to\mathbb{R}\\
&\gamma\mapsto\int_{H^\perp}|\int_B\overline{\gamma(a)\psi(a)}d\mu_A(a)|^{p'}d\mu_{H^\perp}(\psi)
\end{aligned}$$

is continuous.

To show this it is sufficient to check that $q'\circ\Phi:G'\to\mathbb{R}$ is continuous, where $q':G'\to G'/H^\perp$ is the canonical quotient map. Consider a converging net $\gamma_\alpha\to\gamma$ in G'. Since q' is continuous, we have $\gamma_\alpha|_H\to\gamma|_H$ in H'.

Therefore, for a Borel subset K of H with $\mu_H(K)>0$ we have

$$\int_H 1_K(h)\gamma(h)\overline{\gamma_\alpha(h)}d\mu_H(h)\to\int_H 1_K(h)\gamma(h)\overline{\gamma(h)}d\mu_H(h)=\mu_H(K)$$

and

$$\int_G 1_B(a)1_K(h)\gamma(h)\overline{\gamma_\alpha(ah)\psi(ah)}d\mu_G(ah)=$$
$$\Big[\int_B\overline{\gamma_\alpha(a)\psi(a)}d\mu_A(a)\Big]\cdot\Big[\int_H 1_K(h)\gamma(h)\overline{\gamma_\alpha(h)}d\mu_H(h)\Big].$$

Thus, we obtain

$$F_\alpha(\psi) := \int_B \overline{\gamma_\alpha(a)\psi(a)} d\mu_A(a) \to \int_B \overline{\gamma(a)\psi(a)} d\mu_A(a) := F(\psi)$$

for all $\psi \in H^\perp$.

Consider L_2-norms of F and F_α's. Then we have

$$\begin{aligned}
&\|F\|^2_{L_2(H^\perp)} \\
&= \int_{H^\perp} \Big| \int_B \overline{\gamma(a)\psi(a)} d\mu_A(a) \Big|^2 d\mu_{H^\perp}(\psi) \\
(3) \qquad &= \int_{H^\perp} \Big| \int_{G/H} 1_{q(B)}(aH) \overline{\gamma(aH)\psi(aH)} d\mu_{G/H}(aH) \Big|^2 d\mu_{H^\perp}(\psi) \\
&= \int_{G/H} |1_{q(B)}(aH)\overline{\gamma(a)}|^2 d\mu_{G/H}(aH) \\
&= \epsilon > 0
\end{aligned}$$

by Plancherel's theorem.

And similarly we also get $\|F_\alpha\|^2_{L_2(H^\perp)} = \epsilon$. Thus $F_\alpha \to F$ in $L_2(H^\perp)$. Therefore $1_{q(B)}\overline{\gamma_\alpha} \to 1_{q(B)}\overline{\gamma}$ in $L_2(G/H)$. Since $q(B)$ has finite measure, $1_{q(B)}\overline{\gamma_\alpha} \to 1_{q(B)}\overline{\gamma}$ in $L_p(G/H)$. Then by the Hausdorff-Young inequality we get $F_\alpha \to F$ in $L_{p'}(H^\perp)$, and this implies the continuity of Φ.

On the other hand, we assumed that H is discrete, so H' is compact. Thus by the continuity of Φ, the infimum of (2) is attained by an element $\gamma|_H \in H'$. We are expecting that this infimum is positive. Therefore suppose, on the contrary, that this infimum is 0. Then we have

$$\int_{H^\perp} \Big| \int_B \overline{\gamma(a)\psi(a)} d\mu_A(a) \Big|^{p'} d\mu_{H^\perp}(\psi) = 0$$

so that $\int_B \overline{\gamma(a)\psi(a)} d\mu_A(a) = 0$ for almost all ψ. But this is impossible by (3).

If $D_{p'}^{-p'}$ is the infimum of (2), then

$$\begin{aligned}
\left[\int_{H'} \left\|T\hat{f}^H(\gamma|_H)\right\|^{p'} d\mu_{H'}(\gamma|_H)\right]^{\frac{1}{p'}} &\le D_{p'} \left\|T\widehat{g}^G\right\|_{L_{p'}(G',Y)} \\
&\le D_{p'} \|T|\mathcal{F}\mathcal{T}_p\| \cdot \|g\|_{L_p(G,X)} \\
&\le D_{p'}\epsilon^{\frac{1}{p}} \|T|\mathcal{F}\mathcal{T}_p\| \cdot \|f\|_{L_p(H,X)},
\end{aligned}$$

hence we get (1) with $C_p = D_{p'}\epsilon^{\frac{1}{p}}$. □

Example 1. Consider $\mathbb{R}^n$ and $\mathbb{Z}^n$ for a natural number n. Since $\mathbb{R}^n$ is second countable and $\mathbb{Z}^n$ is a discrete closed subgroup of $\mathbb{R}^n$, we have the following:

$$\mathcal{FT}_p^{\mathbb{Z}^n} \subseteq \mathcal{FT}_p^{\mathbb{R}^n}.$$

Remark 2. If we replace the discreteness condition of H by the openness of H, then we get a part of Proposition 1.3 in [1] as follows: Since H is open, $H^\perp$ is compact so that we can set $\mu_{H^\perp}(H^\perp) = 1$ by defining $\mu_H = \mu_G|_H$(see Lemma 3.1 of [1]). Then we have $\|F\|_{L_{p'}} \geq \|F\|_{L_2} = \sqrt{\epsilon}$ by the same argument in (3) in the proof of Theorem 7.

3.2. **The case** $p = 2$**.**

Theorem 8. *Let G be a second countable LCA group and H a closed subgroup of G. Then for any Banach space operator $T : X \to Y$ we have*

$$\left\|T|\mathcal{FT}_2^H\right\| \leq \left\|T|\mathcal{FT}_2^G\right\|.$$

Proof. Since $p = p' = 2$, we already computed the infimum of (2) in the proof of Theorem 7. It is exactly ϵ by (3) in the proof, so we get $D_{p'}\epsilon^{\frac{1}{p}} = \epsilon^{-\frac{1}{p'}}\epsilon^{\frac{1}{p}} = 1$. □

We can show the result of A. Hinrichs [5] in another way using Theorem 8.

Corollary 9. *For any Banach space operator $T : X \to Y$ we have*

$$\|T|\mathcal{WT}_2\| \leq \|T|\mathcal{FT}_2\|.$$

Proof. It is well known that $\|T|\mathcal{FT}_2\| = \left\|T|\mathcal{FT}_2^{\mathbb{T}}\right\| = \left\|T|\mathcal{FT}_2^{\mathbb{T}^\infty}\right\|$ by [3], [1], [10] and $\|T|\mathcal{WT}_2\| = \left\|T|\mathcal{FT}_2^{\mathbb{Z}_2^\infty}\right\|$ by [3]. Since $\mathbb{Z}_2^\infty$ is a closed subgroup of $\mathbb{T}^\infty$ which is second countable, Theorem 8 implies

$$\|T|\mathcal{WT}_2\| \leq \|T|\mathcal{FT}_2\|.$$

□

But unfortunately we can not extend Theorem 8 to the case $p < 2$ in this approach.

Proposition 10. *Let $G = \mathbb{T}^\infty$ and $H = \mathbb{Z}_2^\infty$. Then the infimum of (2) in the proof of Theorem 7 is zero for $p < 2$.*

Proof. Since G and H here are countable products of compact groups, we can naturally consider product probabilities μ_G and μ_H, respectively. And the Borel selection A is $[0, \frac{1}{2})^\infty$ and μ_A is also product probability where the probability on $[0, \frac{1}{2})$ is twice of Lebesque measure by Weil's formula. Thus we can set $B = A$.

Furthermore we can easily check that

$$H' = \{\, b = (b_1, b_2, \ldots) \in \{0,1\}^\infty : b_i = 0, \text{ except for finitely many index } i\text{'s}\}$$

and

$$H^\perp = \{\, c = (c_1, c_2, \ldots) \in \mathbb{Z}^\infty : \text{ all } c_i\text{'s are even and } c_i = 0, \text{ except for finitely many index } i\text{'s}\}.$$

Then we have for $b = (b_1, b_2, \ldots, b_n, 0, 0, \ldots) \in H'$,

$$(2) = \sum_{c \in H^\perp} \Big| \int_{A=[0,\frac{1}{2})^\infty} e^{-2\pi i b_1 \omega_1} \cdots e^{-2\pi i b_n \omega_n} e^{-2\pi i c_1 \omega_1} \cdots e^{-2\pi i c_m \omega_m} d_A(\omega)) \Big|^{p'}$$

(we can assume that $n < m$ by inserting zeros to sequences (c_k)'s.)

$$= \sum_{c \in H^\perp} \Big| \int_{A=[0,\frac{1}{2})^\infty} e^{-2\pi i((b_1+c_1)\omega_1 + \cdots + (b_n+c_n)\omega_n + c_{n+1}\omega_{n+1} + \cdots + c_m\omega_m)} d_A(\omega)) \Big|^{p'}$$

$$= \sum_{c \in H^\perp} \Big\{ \Big[2 \int_0^{\frac{1}{2}} e^{-2\pi i (b_1+c_1)\omega_1} d\omega_1 \Big] \cdots \Big[2 \int_0^{\frac{1}{2}} e^{-2\pi i (b_n+c_n)\omega_n} d\omega_n \Big] \cdot \Big[2 \int_0^{\frac{1}{2}} e^{-2\pi i c_{n+1}\omega_{n+1}} d\omega_{n+1} \Big] \cdots \Big[2 \int_0^{\frac{1}{2}} e^{-2\pi i c_m\omega_m} d\omega_m \Big] \Big\}.$$

Since

$$2 \int_0^{\frac{1}{2}} e^{-2\pi i k t} dt = \begin{cases} 1 & \text{if } k = 0, \\ \frac{2}{k\pi i} & \text{if } k \text{ is odd}, \\ 0 & \text{if } k \text{ is non-zero even} \end{cases}$$

for all integer k, we obtain

$$(2) = \sum_{c_{\sigma(1)}, \ldots, c_{\sigma(l)} \text{ are even}} \Big| \frac{2}{\pi(c_{\sigma(1)}+1)} \Big|^{p'} \cdots \Big| \frac{2}{\pi(c_{\sigma(l)}+1)} \Big|^{p'}$$

$$= \Big[2 \Big(1 + \frac{1}{3^{p'}} + \frac{1}{5^{p'}} + \frac{1}{7^{p'}} + \cdots \Big) \Big(\frac{2}{\pi} \Big)^{p'} \Big]^l,$$

where l is the number of indices i's such that $b_i = 1$ and σ is an injection from $\{1, 2, \ldots l\}$ to $\{1, 2, \ldots, n\}$ such that $b_{\sigma(j)} = 1$.

Since

$$\Big[2 \Big(1 + \frac{1}{3^{p'}} + \frac{1}{5^{p'}} + \frac{1}{7^{p'}} + \cdots \Big) \Big(\frac{2}{\pi} \Big)^{p'} \Big] < \Big[2 \Big(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \Big) \Big(\frac{2}{\pi} \Big)^2 \Big] = 1,$$

the infimum of (2) is zero in this case.

□

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