LOG MINIMAL MODEL PROGRAM OF $\overline{M}_g$ VIA GIT AND STACKS

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These lectures will survey the recent developments in the log minimal model program for the moduli space of curves and discuss Geometric Invariant Theory and stack-theoretic approaches to it.

The first lecture will deal with GIT stability of finite Hilbert points of embedded varieties. We will describe the recent progress on the finite Hilbert stability of general (bi)canonical curves of arbitrary genus [2] and analogous results establishing finite Hilbert stability of certain surfaces.

The generic stability result for curves opens the door to analyzing a whole menagerie of new GIT quotients, which are expected to be log canonical models of the moduli space of stable curves. The study of these models is the subject of a recent program, initiated by Hassett and Hyeon [6, 7], to understand the birational modifications of the moduli space of curves from a functorial point of view. A number of recent papers addresses the log minimal model program for $\overline{M}_g$ in low genera [3, 4, 5, 8, 9] by constructing log canonical models via GIT. The case of genus four and five will be the subject of the second lecture.

The known results in low genera agree with the predictions given in [1] for how the log minimal model program for $\overline{M}_g$ should proceed. In the last lecture, we will discuss the stack-theoretic methods used in this paper and their applications to the construction of the second flip of $\overline{M}_g$.

References

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